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Estimating reliability with discrete growth models

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THESIS

ESTIMATING RELIABILITY WITH
DISCRETE GROWTH MODELS

by

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March 1988

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Estimating Reliability with Discrete Growth Models

by

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Captain, United States Army
B.S., United States Military Academy, 1978

Submitted in partial fulfillment of the
requirements for the degree of

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March 1988

ABSTRACT

Determining the reliability of newly designed systems is one of the most important functions of the acquisition process in the military. Tracking the growth in reliability of a system as it is developed and modified repeatedly is an important part of the acquisition process.

This thesis extends and expands a reliability growth simulation program written previously. It analyzes the capabilities and limitations of two discrete reliability growth models to determine which models are most applicable in estimating system reliability under a variety of different growth patterns. Negative growth patterns are also considered. The result of this thesis is a FORTRAN simulation which enables a more accurate estimate of system reliability using test data generated during the development phase of an acquisition program.

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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I. INTRODUCTION

The reliability of a particular system or piece of equipment is one of the most important factors in assessing its true value. A system with tremendous capabilities that spends an inordinate amount of time in the maintenance facility is usually less desirable than an alternate system with relatively moderate capabilities that has a much higher availability. Therefore, accurately measuring the reliability of a proposed system during the test and design phases is an integral part of the materiel acquisition process. Traditional reliability estimators rely upon conducting a sufficient number of tests or experiments so that a statistically valid point estimate or confidence interval can be established. These traditional estimates assume a constant or fixed reliability during any particular design phase. Since reliability will almost certainly fluctuate from phase to phase of the acquisition process, methods are needed that track reliability as it changes. Additionally, systems being produced presently are more complex and claim a much higher reliability than those produced just a few years ago. Verifying high reliability inherently requires significantly more testing than does verifying relatively moderate reliability since items are tested until failure. Therefore, all test data should be utilized to the utmost.

The process of testing and evaluating newly designed pieces of equipment is very costly in terms of both time and money. Typically, the test engineer must face limitations in both of these resources and occasionally must compromise on either the extensiveness of the test design or the number of trials to be run or both. It may be assumed that the constraints on time and money will not diminish in the near future -- if anything, these constraints will become tighter. Therefore, methods that are able to produce accurate estimates of system reliability for a smaller investment of resources are very desirable. Reliability growth models are one such method.

Reliability growth models make use of all available test data. Results from previous phases of design are combined with current test data so that a pattern of reliability is established. These models often make use of a procedure known as failure discounting. Failure discounting involves removing fractions of previous failures in order to make allowance for reliability improvement as the system evolves. The reliability pattern established by a reliability growth model becomes the basis for producing estimates of the actual system reliability.

There are two types of reliability growth models -- discrete and continuous. A continuous model is based on the time until failure of the system under consideration. Obviously, a number of distributional assumptions are necessary in a model of this type. Discrete reliability growth models are employed when test data references the number of system tests and failures in a particular design phase. In order to use a discrete reliability growth model one must be able to classify a test result as either a failure or a success. In other words, the test data must be attribute data. In this thesis, situations where test parameters may classify a trial as a partial success or a partial failure are not allowed.

This thesis addresses discrete reliability growth models. The major objective is to analyze the capabilities and limitations of two such models. Two different methods of discounting previous failures are also evaluated. A previously designed simulation program [Ref. 1: pp. 36-46] is expanded or modified as necessary in order to more fully develop the two discrete reliability growth models being examined. The original simulation uses actual reliabilities and Monte Carlo techniques in order to generate a random reliability growth pattern. The test data generated by this growth pattern is then used in the reliability growth models in order to produce estimates of system reliability. A variety of different reliability growth patterns and a broad spectrum of failure discounting parameters are systematically evaluated. At each phase, the estimate of system reliability is compared to the actual reliability. The applicability of the two discrete reliability growth models, to include limitations and capabilities is also addressed.

The following chapter describes each of the two discrete reliability growth models being considered. An explanation of the two failure discounting methods is also offered. Additionally, work done in this area previous to this analysis is summarized. Chapter III discusses the methodology used to conduct the analysis, to include the computer simulation. Chapter III also describes the various different growth patterns that were analyzed. Chapter IV presents the results of the analysis. Chapter V details conclusions and recommendations.

II. DISCOUNTING PROCEDURES AND DISCRETE GROWTH MODELS

A. BACKGROUND

In this chapter, two discrete reliability growth models are described. Two separate techniques of discounting failures are also explained. Both methods remove a fraction of a previous failure when the system successfully completes a trial without a reoccurrence of the particular failure cause. Reliability is defined as the probability of the system successfully completing a single trial. The exact definition of a test trial is much more difficult to fix since, typically, the constitution of a trial is the responsibility of the testing agency. Normally, a trial may be considered as the exercising of the particular system in a manner consistent with its purpose. If a weapons system is being evaluated, for example, then a trial could consist merely of a single attempt at target acquisition or proceed through target acquisition to an attempt at target destruction. The precise definition of a trial depends in large measure on the purpose or goal of the test. For the purpose of this thesis and in order to utilize discrete reliability growth models, a trial must be defined so that it is discrete and can be evaluated as either a success or failure. Partial successes are not allowed.

A testing phase may be defined as a number of trials, one or more, during which the configuration of the system is unchanged. Therefore, during a testing phase the actual system reliability remains unchanged. A test phase may consist of one or more system failures. If improvements or changes to the system are effected after each system failure (a test-fix-test scenario) then a phase would consist of one system failure and all of the successful trials leading up to that failure. Alternatively, the test design may dictate that testing be continued until a certain number of failures occur. Under this scenario (test-find-test) the cause of system failure is identified but system configuration remains unchanged until the pre-designated failure occurs. In general, a phase will encompass all of the trials between system configuration changes.

B. FAILURE DISCOUNTING

Testing conducted during the initial design stages of a particular system often indicates low reliability. Generally, weaknesses in the configuration of the system or defects in the quality of its components cause system failure. Test designs are established so that the cause for these failures can be identified and corrected. Theoretically, then, as a weakness or a defect is identified and, hopefully, corrected the probability of that particular weakness or defect reoccurring should be reduced. This reduction in the probability of occurrence of a certain failure cause leads to improved system reliability. This concept is fully utilized in failure discounting.

In order to effectively discount previous failures it is critical that the cause of the failure be properly identified. The level of detail that one wishes to ascribe to this identification process is dependent upon the type of system being evaluated and the purpose of the test. If a complex system is being evaluated then a failure cause may be failure of a certain component or sub-component. The precise element that caused system failure is not critical but the ability to assign a failure cause to each system failure is. Correctly determining failure cause is very difficult, particularly when dealing with complex systems. Therefore, it is conceivable that design changes do not improve system reliability -- in fact, these changes may even degrade reliability. To apply the failure discounting procedures described below, one must be able to assign a failure cause to every system failure even though it may be done erroneously.

1. Standard or Straight Percent Failure Discounting

The standard discounting method will reduce a previous failure by a fixed amount after a predesignated number of trials have been successfully conducted without a reoccurrence of a failure due to the same cause. This method has been detailed fully in previous work [Ref. 1: pp. 14-17] so only a brief description and an example of its application will be included here.

To employ the standard discounting method one must specify two parameters - the fraction of a failure to be removed, F, and the discount interval or the number of successful trials, I, that must occur before application of the discount. For the purpose of this paper and to use the simulation described herein, these parameters are specified at the outset and remain constant throughout the test. The number of successful trials since the occurrence of the failure is referred to as T. These three values, F, I, and T are then used to discount previous failures at the end of each phase.

Mathematically, the standard discounting method is given by Equation 2.1:

$$DISCOUNTED\ FAILURE = (1 - F)^{INT(\frac{T}{I})} \qquad (2.1)$$

The term INT in the above equation refers to the integer portion of the argument $(\frac{T}{I})$. As such, if the number of successful trials, since the failure cause last occurred, T, is less than the number of trials required by the discount interval, I, then $INT(\frac{T}{I})$ will be zero thus causing no discounting to be applied. Additionally, should a failure cause reoccur then the value of T is returned to zero thus removing any previous discounting that may have been applied. This ability, in effect, acts as a penalty factor. If a failure cause should reoccur then previous design changes may not have been effective and thus any prior failure discounting could be unwarranted. The rate and amount of failure discounting can be controlled by altering F and I. If the system under evaluation is relatively simple and a proven failure identification process is in place, then a large discount to system failures may be warranted. This will result in a prediction of rapid reliability growth.

Table 1. STANDARD DISCOUNT METHOD EXAMPLE

PHASE	TRIAL	TOTAL FAILURES	CAUSE	
			X	Y
1	1	0	S	S
1	2	1	F	S
2	3	1	S	S
2	4	1	S	S
2	5	2	S	F
3	6	2	S	S
3	7	2	S	S
3	8	2	S	S
3	9	2	S	S
3	10	2	S	S
3	11	2	S	S
3	12	3	F	S

A small example of the application of the standard discount method will aid in clarifying its abilities. Consider the data in Table 1. This test data is purely fictitious and is intended solely to illustrate the use of the standard discount method. We will assume that improvements or repairs are effected after each failure so actual system reliability is not constant.

Table 1 represents three phases of a test-fix-test scenario where two failure causes, X and Y, have been identified. It should be noted that under a test-fix-test plan, a phase will terminate with system failure. The result of each trial is listed as either a success or failure for a particular failure cause.

The two parameters necessary to employ the standard discount method are the fraction of failure to be removed, F, and the discount interval, I. Assume that F = 0.50 and I = 3. System failure number two, attributed to failure cause Y, terminates phase two. At that point the failure due to cause X that terminated the first phase has had three successive successful trials so failure discounting can be applied:

$$ADJUSTED\ FAILURE = (1 - 0.50)^{(\frac{3}{3})} = 0.50$$

Thus, for reliability computations at the conclusion of the second phase, the failure that occurred during phase 1 is only counted as one-half of a failure.

Phase three terminates with a failure due to cause X. The value for T then is set to zero and thus when applied to the failure that occurred in phase one, full value is restored. At the end of phase two this failure had been discounted to 0.50 but, because the failure cause reoccurred, this discount is viewed as unwarranted and the failure is returned to one. However, at the end of phase three failure cause Y has had seven successful trials. Therefore, discounting is applied:

$$ADJUSTED\ FAILURE = (1 - 0.50)^{INT(\frac{7}{3})} = (0.50)^2 = 0.25$$

Therefore, when computing reliability estimates at the end of phase three, only 2.25 system failures (failure number one plus failure number three plus the discounted value of failure number two) are considered instead of the three failures that actually occurred.

The standard discount method described is very flexible because of its two parameters. Setting values for F and I does, however, require a good deal of professional judgement and expertise. To avoid conflicts in setting such values David K. Lloyd proposed an alternate method of discounting failures [Ref. 2]. The Lloyd method is described in the next section.

2. Lloyd Failure Discounting

Discounting previous failures using the Lloyd discounting method is based on the premise that there should be some sort of statistical basis for determining how much a previous failure should be discounted. Lloyd offers the upper confidence limit of the probability that a failure cause will occur again as the discounted value of its respective failure. The confidence limits for this probability are based on the number of successful trials since the failure cause last occurred. Under a test-fix-test scenario this is equivalent to the number of successful trials since the last failure. This number was defined in the description of the standard discount method as T. In order to implement Lloyd failure discounting, one must set the level of confidence desired for the confidence bound. Since the method is applied on every trial there is no requirement to specify an interval, although as will be explained in a later chapter, specifying an interval for the Lloyd method can be done and may be advantageous in some situations.

The Lloyd discount method may be mathematically expressed as follows:

$$ADJUSTED\ FAILURE = 1 - (1 - CI)^{\frac{1}{T}} \quad if\, T > 0 \tag{2.2}$$

In the above equation CI is defined as the level of confidence desired and T is the number of successful trials. If T has a value of zero then the value of the adjusted failure is set to one. Setting the value of the adjusted failure equal to one when T equals zero gives the Lloyd method the ability to restore all of the value of a discounted failure should its failure cause reoccur.

Table 2. LLOYD DISCOUNT METHOD EXAMPLE

PHASE	TRIAL	TOTAL FAILURES	FAILURE CAUSE		ADJUSTED FAILURES
			X	Y	
1	1	0	S	S	0
1	2	1	F	S	1.000
2	3	1	S	S	0.900
2	4	1	S	S	0.684
2	5	2	S	F	0.536 + 1.000 = 1.536
3	6	2	S	S	0.438 + 0.900 = 1.338
3	7	2	S	S	0.369 + 0.684 = 1.053
3	8	2	S	S	0.319 + 0.536 = 0.855
3	9	2	S	S	0.280 + 0.438 = 0.718
3	10	2	S	S	0.250 + 0.369 = 0.619
3	11	2	S	S	0.226 + 0.319 = 0.545
3	12	3	F	S	2.000 + 0.280 = 2.280

Table 2 illustrates the use of the Lloyd method. The test data is the same data used to demonstrate the standard discount method. Since the Lloyd method is applied on every trial a fourth column has been added to the table reflecting the current value of the adjusted failure. Let us assume that the confidence interval desired in this case was .90.

If reliability is computed at the completion of the third phase then 2.28 system failures would be used under the Lloyd discounting method as opposed to the actual value of three that occurred. The last column of the table indicates the speed that failures become discounted with the Lloyd method.

The two discrete reliability growth models that will be examined are described in the next section. Each model makes use of discounted failure data and each has demonstrated, through simulation, the ability to estimate constant and concavely increasing reliability with reasonable accuracy.

C. DISCRETE RELIABILITY GROWTH MODELS

Both of the discrete reliability growth models to be presented assume a constant system reliability within a phase. The first model to be examined will be the Maximum Likelihood Estimate with Failure Discounting (MLEFD). This model is merely a derivative of the conventional single phase maximum likelihood estimate. The single phase maximum likelihood estimate considers only test data generated during the current phase. The MLEFD, conversely, is cumulative in that it considers all of the available test data. The test data from early phases during which the actual system reliability was different requires adjustment. Thus, the use of failure discounting techniques provides a method by which failures occurring early in the development process can be assimilated into current reliability estimates. The second model is a regression model based on an exponential single phase reliability estimate. It estimates reliability for each phase using an exponential model and then performs a linear regression on these estimates to obtain a current estimate. The simulation program that was written to evaluate these two models was designed so that failure discounting may or may not be invoked with either model.

Before offering a general description of the two models it is important at this juncture to digress for a moment and discuss the exact methodology that will be used to incorporate discounted failures into the various reliability estimates. As was seen in the section prior, discounted or adjusted failures are rarely if ever integer-valued. While this does not present a problem for the MLEFD reliability growth model it does pose an obstacle to using the exponential regression model. The convention devised in Captain Drake's thesis [Ref. 1: p. 23] will be continued in this paper. The number of trials up to and including a particular failure will be divided by the adjusted failure so that an adjusted number of trials is computed. The adjusted failure value is then returned to one from its previous fractional value. The formula for computing adjusted trials is then:

$$ADJUSTED\ TRIALS = \frac{TOTAL\ TRIALS}{ADJUSTED\ FAILURE} \tag{2.3}$$

The appeal of this method is that the result is an integer number of failures while maintaining a constant ratio of number of failures to number of trials. While the adjusted number of trials may not be integer-valued they may be adjusted to integers, if necessary, through rounding with a much smaller effect on reliability computations than rounding adjusted failures.

If a failure had been discounted (using either discounting method) from one to .25, for example, and the number of successful trials since the failure occurred was ten then the ratio of failures to successes would be .25 to ten. Using the method described above, the adjusted failure and actual trials are divided by the adjusted failure yielding a failure to success ratio of one to forty. Thus, the number of trials has been adjusted from ten to forty and the number of failures has been returned to one from .25. Since the ratio of failures to successes remains the same reliability computations will not be affected. Both of the models described below will make use of this method in computing reliability estimates.

1. Maximum Likelihood Estimate with Failure Discounting

The traditional estimate of system reliability is the maximum likelihood estimate. Letting $\hat{R}$ denote the estimate of reliability, the maximum likelihood estimate may be expressed as follows:

$$\hat{R} = \frac{TOTAL\ TRIALS - TOTAL\ FAILURES}{TOTAL\ TRIALS}$$

or alternatively:

$$\hat{R} = \frac{SUCCESSFUL\ TRIALS}{TOTAL\ TRIALS}$$

As stated previously, the maximum likelihood estimate assumes a constant reliability between phases. It also requires a large number of trials to accurately estimate the underlying system reliability. Failure discounting is used to make previous test data compatible with current results and thus increase the total number of trials available. However, since the actual system reliability is unknown it becomes a very non-trivial task to select the appropriate discounting method and the correct parameters.

When previous test data is made available through the use of a discounting routine the equation for estimating the system reliability becomes:

$$\hat{R} = \frac{TOTAL\ TRIALS - TOTAL\ ADJUSTED\ FAILURES}{TOTAL\ TRIALS}$$

or if the convention of restoring the number of failures to integer values is adopted

$$\hat{R} = \frac{TOTAL\ ADJUSTED\ TRIALS - TOTAL\ FAILURES}{TOTAL\ ADJUSTED\ TRIALS} \tag{2.4}$$

To utilize the MLEFD reliability growth model one first records all of the test data from the current phase. The discounting method to be used is then performed on the previous test data and an adjusted failure value is computed for each previous failure. The adjusted number of trials for each failure is then computed using Equation 2.3. The estimate of reliability for the current phase is then calculated using Equation 2.4 above.

One deficiency of the maximum likelihood estimate is that, when testing is terminated after a fixed number of failures, the MLE is inherently a biased estimator of the actual reliability. The expected value of the maximum likelihood estimate has been derived in previous work [Ref. 3: p.34]. It may be expressed mathematically as follows:

$$E[\hat{R}] = 1 + (\frac{1}{R})\ln(1-R) - \ln(1-R) \tag{2.5}$$

where R represents the actual reliability of the system. The bias, $B(\hat{R})$, in the estimate may be calculated using the formula:

$$B(\hat{R}) = E[\hat{R}] - R$$

Table 3 below depicts the performance of the maximum likelihood estimate for different values of actual reliability, R. The bias is inherent to the estimate in that it will be present regardless of the validity of the assumption of constant phase reliability.

The use of failure discounting can offset the effect of bias as well as handle changing system reliability. However, this puts a lot of additional weight on the ability to correctly select the best method and choose the proper parameters. Alternatively, if an estimator of system reliability had to be biased one would want it to be conservatively biased as is the MLE. Underestimating actual system reliability is normally less costly than overestimating. However, a less biased estimate would be beneficial.

Table 3. INHERENT BIAS IN THE MAXIMUM LIKELIHOOD ESTIMATE

RELIABILITY	$E[\hat{R}]$	$B(\hat{R})$
0.50	0.307	-0.193
0.60	0.389	-0.211
0.70	0.484	-0.216
0.80	0.598	-0.202
0.90	0.744	-0.156
0.95	0.842	-0.108
0.97	0.892	-0.078
0.99	0.953	-0.037

2. Exponential Regression Estimate

The exponential regression reliability growth model was developed by H. Chernoff and W.M. Woods and based on the exponential single phase estimate [Ref. 4]. The derivation of this model has been developed fully in previous work [Ref. 5: pp. 3-5] so for the purposes of this paper a general summary of results will be offered.

The basic form of the exponential single phase estimate for a phase k i.e., after the kth improvement or alteration has been made, is stated in Equation 2.6:

$$\tilde{R}_k = 1 - e^{(-Y_k)} \tag{2.6}$$

where $\tilde{R}_k$ is the reliability estimate and the coefficient of the exponential term, Y, can be calculated as follows:

$$Y_k = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{(X_k - 1)} \qquad \text{if } X_k \geq 2$$

where X_k is the number of trials up to and including the first failure in phase k. If X_k equals one then Y_k is defined equal to zero. It should be noted that this estimate was developed for the special case of testing until the first failure is encountered. If system failure occurs on the first trial ($X_k = 1$) then Y_k will equal zero and system reliability will be estimated as $1 - e^0 = 1 - 1 = 0$. If failure occurs on the third trial, then the exponential single phase estimate would yield $1 - e^{-(1+\frac{1}{2})} = 0.777$.

In order to evaluate the properties of the exponential single phase estimate a small simulation was designed. Geometric test data with actual reliability R were generated using a transformation of a standard Uniform(0,1) random number. Ten thousand sets of geometric data were generated for each value of R. Table 4 represents the results of this simulation. Also included in the table is a simulation of the maximum likelihood estimate, ($\hat{R}$), using the same test data.

The values generated for the mean of the maximum likelihood estimate, $\hat{R}$, conform fairly well with the theoretical results, thus the simulation itself is accurate. The values generated for the exponential estimate, $\tilde{R}$, while still underestimating the actual reliability, are clearly less conservatively biased than the maximum likelihood estimates. The sample variance and mean square error of each estimate is also represented in the table. The mean square error of each estimate is the sum of the variance and the square of the bias. It can be seen that as the actual reliability exceeded .70 the mean square error for the exponential estimate became less than the mean square error of the maximum likelihood estimate. Therefore, the advantage of a lower variance for the maximum likelihood estimate at values of actual reliability below .90 is offset by the greater bias.

Table 4. **PROPERTIES OF THE EXPONENTIAL RELIABILITY ESTIMATE**

R	$E[\hat{R}]$	Maximum Likelihood Estimate			Exponential Estimate		
		$\overline{\hat{R}}$	$\overline{Var(\hat{R})}$	$\overline{MSE(\hat{R})}$	$\overline{\tilde{R}}$	$\overline{Var(\tilde{R})}$	$\overline{MSE(\tilde{R})}$
0.50	0.307	0.302	0.101	0.141	0.357	0.137	0.157
0.60	0.389	0.387	0.111	0.157	0.451	0.143	0.166
0.70	0.484	0.480	0.115	0.163	0.547	0.140	0.163
0.80	0.598	0.595	0.107	0.149	0.661	0.121	0.140
0.90	0.744	0.743	0.079	0.103	0.797	0.080	0.090
0.95	0.842	0.842	0.050	0.062	0.881	0.047	0.052
0.97	0.892	0.892	0.034	0.040	0.921	0.031	0.033
0.99	0.953	0.954	0.014	0.015	0.968	0.012	0.012

The exponential regression reliability growth model developed by Chernoff and Woods uses the technique of linear regression to estimate the coefficient of the exponential term. The model can be expressed mathematically as follows:

$$\tilde{R}_k = 1 - e^{-(\alpha+\beta k)} \tag{2.7}$$

In the equation above k denotes the testing phase being used to compute the system reliability. α and β are computed at the end of each phase using the techniques of linear regression thus yielding an estimate of system reliability. The exact derivation of the linear regression formula for α and β is detailed in other sources [Ref. 3: p. 3] and is provided at Appendix A for the interested reader.

This model requires that both failures and trials be integers. Therefore, when adjusted trials are computed using Equation 2.3, the value obtained must be rounded to the nearest integer. Since linear regression is employed, this model has the ability to track changing reliability without the use of failure discounting.

D. SUMMARY OF PREVIOUS WORK

Analysis of reliability growth models has been a continuing process. and many advances have been made in determing their properties [Refs. 6,7]. It would require many pages to summarize all of the previous work in this area. For the purposes of this paper. the summary will detail only those results which have a direct bearing on the simulation being used to evaluate the two reliability growth models.

A simulation to evaluate these models was constructed by Captain James Drake [Ref. 1: pp. 36-46]. This simulation was written to handle growth patterns where actual reliability was constant or growing at a fixed rate. Since one of the objectives of this paper is to expand the number of reliability growth patterns that can be simulated it is important, at this time, to summarize Captain Drake's work to include the assumptions necessary to run the simulation and some of the limitations.

Many reliability estimators are too mathematically complex to allow closed form solutions of their properties. In order to compare these properties computer simulation is one of the only techniques available. The simulation constructed by Captain Drake builds a reliability growth pattern and generates test data based on known system reliabilities using the geometric distribution. The discrete reliability growth model being evaluated has access to this test data for reliability estimation. The test data generated by the computer simulation is typically the number of trials up to and including failure and the cause of system failure for each predesignated failure in a phase.

The inputs required to run the original simulation are [Ref. 1: pp. 40-41]:

1. Number of testing phases.
2. Number of system failures allowed in each phase.
3. Number of possible failure causes.
4. Probability of non-occurrence of a particular failure cause in the first phase.
5. Reliability growth fraction.
6. Discounting option and parameters.

Several assumptions were necessary to produce an adequate simulation based on these inputs. These assumptions are grouped into two categories -- reliability growth pattern assumptions and failure cause assumptions. The assumptions required for simulating the reliability growth pattern are [Ref. 1: pp. 37-38]:

1. The reliability growth pattern is non-decreasing.
2. System reliability changes only at phase boundaries.
3. Equipment improvements are implemented immediately after a phase ends and before any further testing.
4. Failure causes are corrected only at the end of a phase.
5. Each design improvement or repair removes a fixed fraction of the probability of reoccurrence for the corresponding failure cause.

These assumptions narrowed the scope of the reliability growth patterns that could be evaluated. The first assumption does not allow the model to track any reliability growth patterns that exhibit periods of declining reliability. While for the most part actual system reliabilities will be non-decreasing, this restriction does preclude a complete evaluation of the reliability estimators under consideration. Assumptions two and three are consistent with the previously stated definition of a testing phase. Assumption three does not allow modeling of long term design changes that while indicated, may be postponed for whatever reason. Assumption five represents the method that is used in the simulation to model the impact of design changes or improvements. According to this assumption, any improvement or design change will remove a fixed fraction (user input number five) of the probability of occurrence for a failure cause. The underlying assumption is that all improvements are equally effective.

These assumptions were necessary to generate the type of reliability growth patterns desired in Captain Drake's thesis. By introducing the concept of fixed phase reliability and modifying the original simulation accordingly, four of these five assumptions were no longer necessary. The exact methodology used to effect this modification will be detailed in the next chapter.

The last category of assumptions in the original simulation are the failure cause determination assumptions. These assumptions are [Ref. 1 : p. 39]:

1. There exists a finite number of possible failure causes.
2. Each failure cause has a fixed probability of occurrence in each phase.
3. System reliability can be modeled as a series system of the failure causes.
4. Each failure cause is stochastically independent of the other failure causes.

Assumption three dictates the method by which the actual system reliability is computed in the simulation. Since the occurrence of a failure cause, by definition, means system failure, modeling the system as a series is acceptable. This implies then that the actual system reliability may be mathematically expressed as:

$$R = \prod_{i=1}^{n}(1 - P_i) \tag{2.8}$$

In the above equation P_i is the probability of occurrence of the ith failure cause and n is the total number of failure causes.

The simulation is able to generate descriptive statistics about each of the reliability growth models being examined. These statistics include the mean, its confidence interval and the standard deviation for each reliability estimate at each phase. In order to produce these statistics the simulation should be replicated a sufficient number of times. The mean and the standard deviation are computed in the usual manner. The confidence interval calculation assumes a normal distribution. For the purposes of this paper, 500 replications of each set of input data were conducted so the normality assumption should be reasonable.

One of the limitations of the model in addition to the restricted number of reliability growth patterns that could be analyzed was in the application of the discounting methods. The full range of input parameters could not be explored due, in large measure, to computer limitations. This limitation typically manifested itself at relatively high system reliability. If a failure cause with a very low probability of occurrence caused system failure early and if the input parameters for the discounting method were set such that a relatively large fraction of this failure was discounted over a relatively short number of successful trials then it was possible for this early failure to be discounted to essentially zero. When the adjusted trials were calculated utilizing Equation 2.3 ($ADJUSTED\ TRIALS = \frac{ACTUAL\ TRIALS}{ADJUSTED\ FAILURE}$), then the value approached infinity creating a system error in the computer.

This problem was solved by specifying a lower bound for the adjusted failure. If the failure was discounted so that its adjusted value was less than .0000001 then the adjusted failure was set to .0000001. This modification allowed the full range of input parameters for the discounting methods to be evaluated. The effect on the estimate of system reliability is negligible.

III. METHODOLOGY

One of the major objectives of this paper is to expand the simulation described in the previous chapter so that a greater variety of reliability growth patterns could be used to evaluate the two reliability growth models. This was accomplished through a modification of the simulation that allowed the user to input the desired system reliabilities at each phase. This chapter will detail the method that was used to effect this modification along with a description of the reliability growth patterns and failure discounting options that were used.

A. FIXED PHASE RELIABILITY

One of the limitations of the original simulation was that only constant reliability or reliability that increased at a constant rate could be modeled. If a predesignated failure cause resulted in a failure then that failure cause's probability of occurrence on any succeeding phase was reduced by a predetermined constant amount. Thus, once the user of the simulation established the initial inputs for phase one, he had very limited control over the actual system reliabilities in the subsequent phases.

In order to evaluate the two reliability growth models under a greater variety of potential reliability growth patterns the concept of fixed phase reliability was introduced as an option to the original simulation. With this modification of the program, the user has the ability to fix the actual system reliability at each phase. Thus, the reliability growth models can be used to estimate system reliability under any reliability growth pattern the user desires.

Incorporating fixed phase reliability into the simulation necessitated an expansion of the user-provided input. The basic premise used in computing actual system reliability remains the same. This premise is that system reliability is the product of the probabilities of success of the failure causes (Equation 2.8, $R = \prod_{i=1}^{n}(1 - P_i)$). Originally, the simulation required that the user input the probability of success for each failure cause for the first phase. After that, the fractional improvement factor dictates the actual system reliability in succeeding phases. If the option of fixing the phase reliabilities is chosen, then the user must specify the probability of success for each failure cause in each phase. Equation 2.8 is then applied at each phase to yield the system reliability.

The table below illustrates the inputs that would be required to produce a growth pattern that initially declines and then increases rapidly. The scenario depicted is a simple one involving five test phases with one failure per phase and two failure causes, X and Y. Desired reliability is user determined.

Table 5. FIXED PHASE RELIABILITY EXAMPLE

PHASE	DESIRED RELIABILITY	USER INPUTS		ACTUAL RELIABILITY
		PROBABILITY OF SUCCESS		
		CAUSE X	CAUSE Y	
1	0.80	0.90	0.89	0.801
2	0.70	0.83	0.84	0.697
3	0.65	0.77	0.84	0.647
4	0.75	0.84	0.89	0.748
5	0.90	0.94	0.96	0.902

The actual reliability for each phase was computed using Equation 2.8. Consider phase one, for example. The desired system reliability is 0.80. To achieve this reliability, the probabilities of success for failure causes X and Y were input as 0.90 and 0.89 respectively. This yielded a computed system reliability of:

$$R = \prod_{i=1}^{n}(1 - P_i) = .90 \times .89 = 0.801$$

Figure 1 is a graphical representation of the reliabilities computed in the table. The original simulation could not have produced this reliability pattern since declining reliability is present. This scenario may occur for systems during a portion of their development. While it is true that, in the long run, most systems exhibit non-decreasing reliabilities it is important to consider decreasing reliability when evaluating growth models.

In order to invoke this ability to specify reliability at each phase, the number of user provided inputs increased from two to ten. If there had been two failures allowed in each of the five testing phases then the user inputs would have increased from ten to twenty. Therefore, while the option of allowing the user of the simulation to control the actual system reliability at each phase significantly increases the variety of different reliability

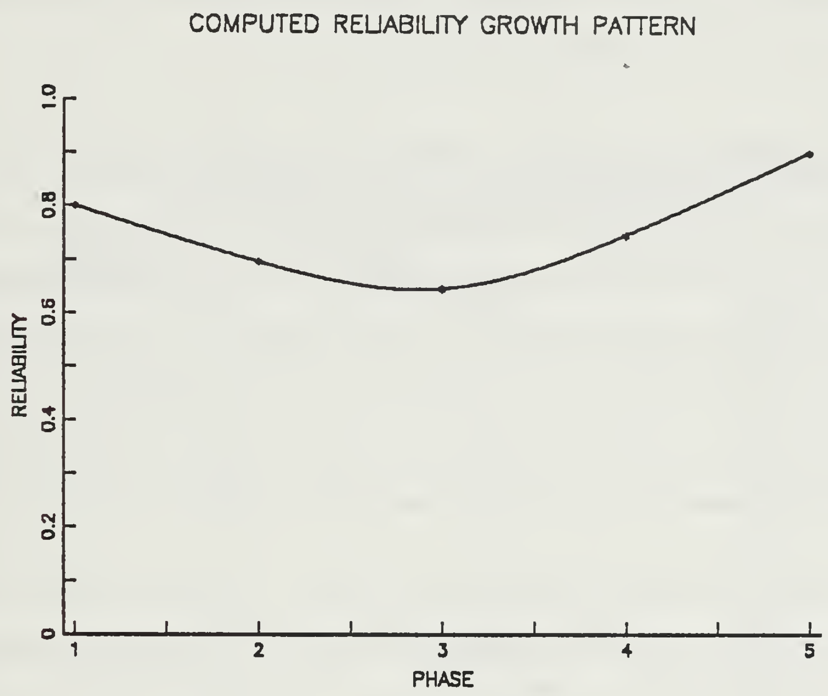

Figure 1. Fixed Phase Reliability Example

growth patterns that can be analyzed, a rather large increase in the volume of input data is required.

B. RELIABILITY GROWTH PATTERNS

To more fully evaluate the properties and trends of the two reliability growth models described in the previous chapter, it was necessary to analyze their performance with respect to a variety of different reliability patterns. The eight growth patterns that were analyzed in this paper will be presented in this section.

In conjunction with the reliability growth patterns, another important impact on the performance of the reliability estimators was the choice of discounting method. The table below depicts the various combinations of parameters for the standard discount method that were analyzed. The discount fraction is the amount of a previous failure that is removed each time the discount method is applied while the discount interval is the number of successful trials between applications. The Lloyd discount method only has one parameter, the confidence interval. For the purposes of this paper, the Lloyd method was applied with confidence values of .8 and .9.

Table 6. STANDARD DISCOUNT METHOD COMBINATIONS

STANDARD DISCOUNT METHOD	
DISCOUNT FRACTION	DISCOUNT INTERVAL
0.00	--
0.25	1
0.25	3
0.25	6
0.25	15
0.50	3
0.50	6
0.50	15
0.75	3
0.75	6
0.75	15

The scenarios used to analyze the reliability growth models in this paper involved ten testing phases, i.e., testing was conducted until ten changes had been made. A total of five failure causes were considered in each test and a limit of one failure per phase was established. The simulation is able to handle any number of test phases, any number of failure causes, and any number of failures per phase. The only limitation is the capacity of the computer on which the simulation is run. Another practical limitation involving the fixed reliability option is the volume of input data that would accompany a test design involving numerous phases or failure causes. As can be seen by the reliability growth patterns and user input tables following, the probabilities of success for the failure causes were allowed to fluctuate. The only requirement was that the reliability growth pattern established accurately reflected the desired growth pattern.

Figure 2 and Table 7 describe the first reliability growth pattern. This is not a very conventional pattern in that most estimators assume concave growth patterns of the form shown in Figures 5 and 6. However, this type of reliability growth pattern would not be unusual in situations where the exact method or technology required to correct a failure-causing defect is not immediately available but, as the system evolves and personnel become more familiar with it, the the failure correction process proceeds more efficiently.

Figure 3 depicts a decreasing reliability growth pattern. In this instance, an attempted improvement actually caused system reliability to decrease. This situation, which is not effectively addressed by any conventional reliability estimators. could possibly occur in experimental or technologically advanced systems where the complete ramifications of design changes or corrections are not known.

Figure 4 represents a scenario in which the system under consideration attains a moderately high reliability and then stagnates at that level for several phases before starting to improve again. This scenario could occur if the exact cause of system failure is difficult to assess and would not be uncommon if the system being developed is highly complex.

Figures 5 and 6 depict conventional reliability growth patterns one would expect to encounter when evaluating the majority of systems. In Figure 5, the system reliability increases rapidly to 0.99 and then remains constant whereas, in Figure 6, the system reliability increases rapidly to 0.90 before becoming constant. It is important in the evaluation of the reliability growth models to compare their performance in relatively atypical, although not uncommon, scenarios such as those shown in Figures 2 through 5 with their performance in more conventional situations.

Figures 7, 8, and 9 describe a constant system reliability of 0.90, 0.60, and 0.40 respectively. While it would be rare to find these system reliabilities occurring in actual practice, their inclusion will illuminate some of the trends of the growth models under the various failure discounting methodologies.

Each reliability growth pattern was simulated 500 times for each of the failure discounting combinations described previously. This will enable a more accurate appraisal of the capabilities and limitations of the two reliability growth models being considered. In addition, the most commonly used estimator of reliability, the single phase maximum likelihood estimate, will be included on all figures so that a basis for comparing the two reliability growth estimators exists. The single phase maximum likelihood estimate, in contrast to the maximum likelihood estimate with failure discounting being evaluated in

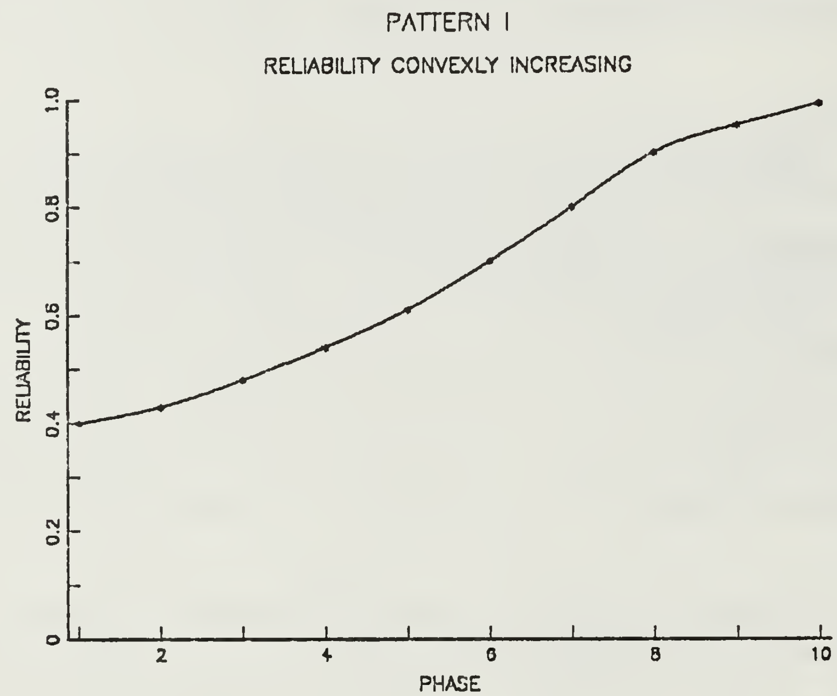

Figure 2. Pattern I

Table 7. PATTERN I USER INPUTS

CAUSE	PHASE PROBABILITY OF SUCCESS									
	1	2	3	4	5	6	7	8	9	10
1	.85	.86	.90	.91	.93	.95	.97	.99	.99	.998
2	.84	.85	.87	.90	.92	.95	.97	.99	.99	.998
3	.83	.84	.86	.88	.90	.93	.96	.98	.99	.998
4	.83	.84	.85	.87	.89	.92	.94	.975	.99	.998
5	.81	.83	.84	.86	.89	.91	.94	.961	.99	.998

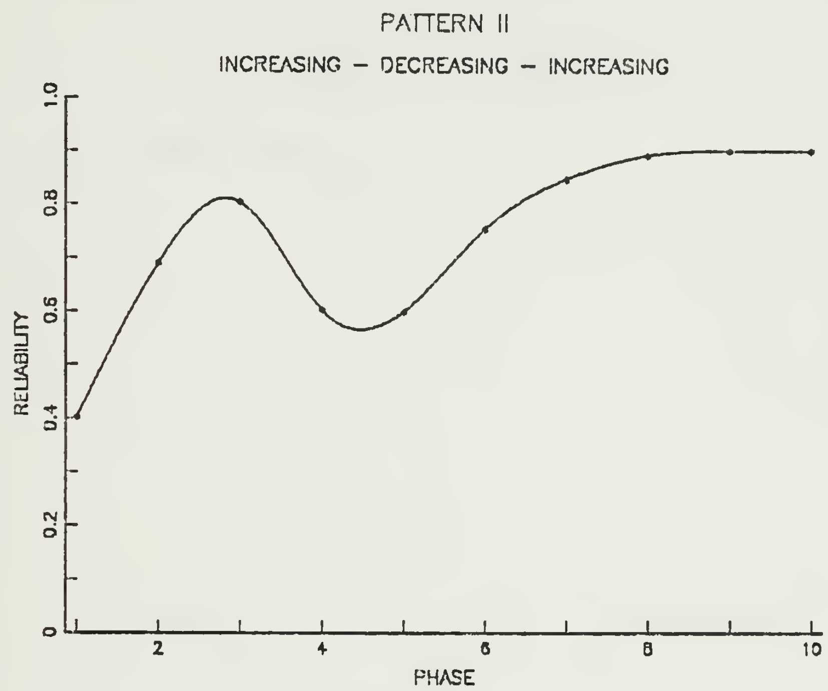

Figure 3. Pattern II

Table 8. PATTERN II USER INPUTS

CAUSE	PHASE PROBABILITY OF SUCCESS									
	1	2	3	4	5	6	7	8	9	10
1	.98	.98	.99	.99	.99	.99	.99	.99	.99	.99
2	.95	.97	.98	.98	.78	.93	.98	.99	.99	.99
3	.82	.93	.96	.72	.90	.95	.97	.98	.98	.98
4	.80	.92	.96	.96	.96	.96	.96	.97	.98	.98
5	.66	.85	.90	.90	.90	.90	.94	.96	.96	.96

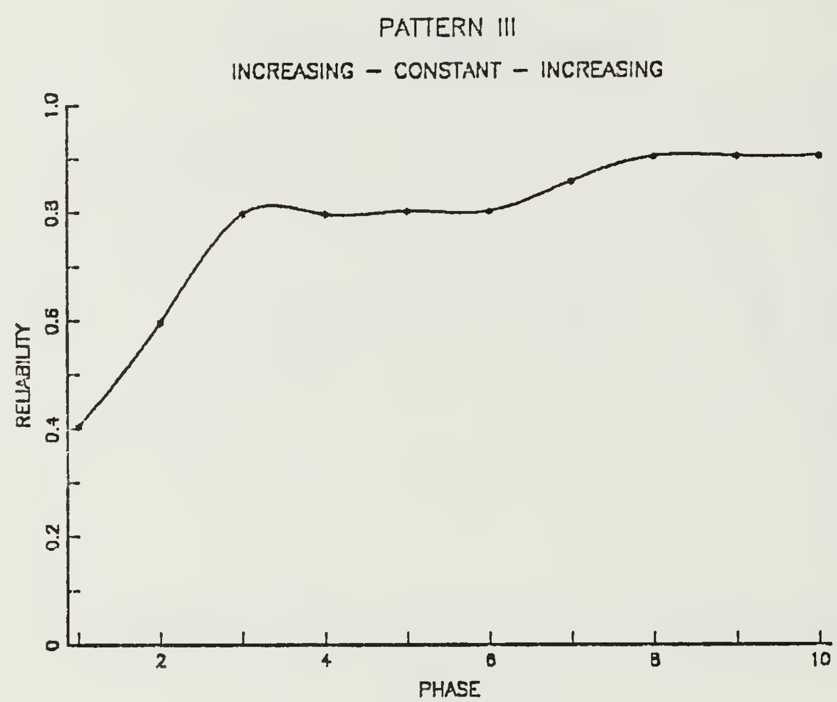

Figure 4. Pattern III

Table 9. PATTERN III USER INPUTS

CAUSE	PHASE PROBABILITY OF SUCCESS									
	1	2	3	4	5	6	7	8	9	10
1	.86	.93	.97	.97	.98	.98	.98	.99	.99	.99
2	.86	.93	.97	.97	.98	.98	.98	.99	.99	.99
3	.86	.93	.97	.97	.98	.98	.98	.99	.99	.99
4	.86	.93	.97	.97	.98	.98	.98	.99	.99	.99
5	.74	.80	.90	.90	.87	.87	.93	.94	.94	.94

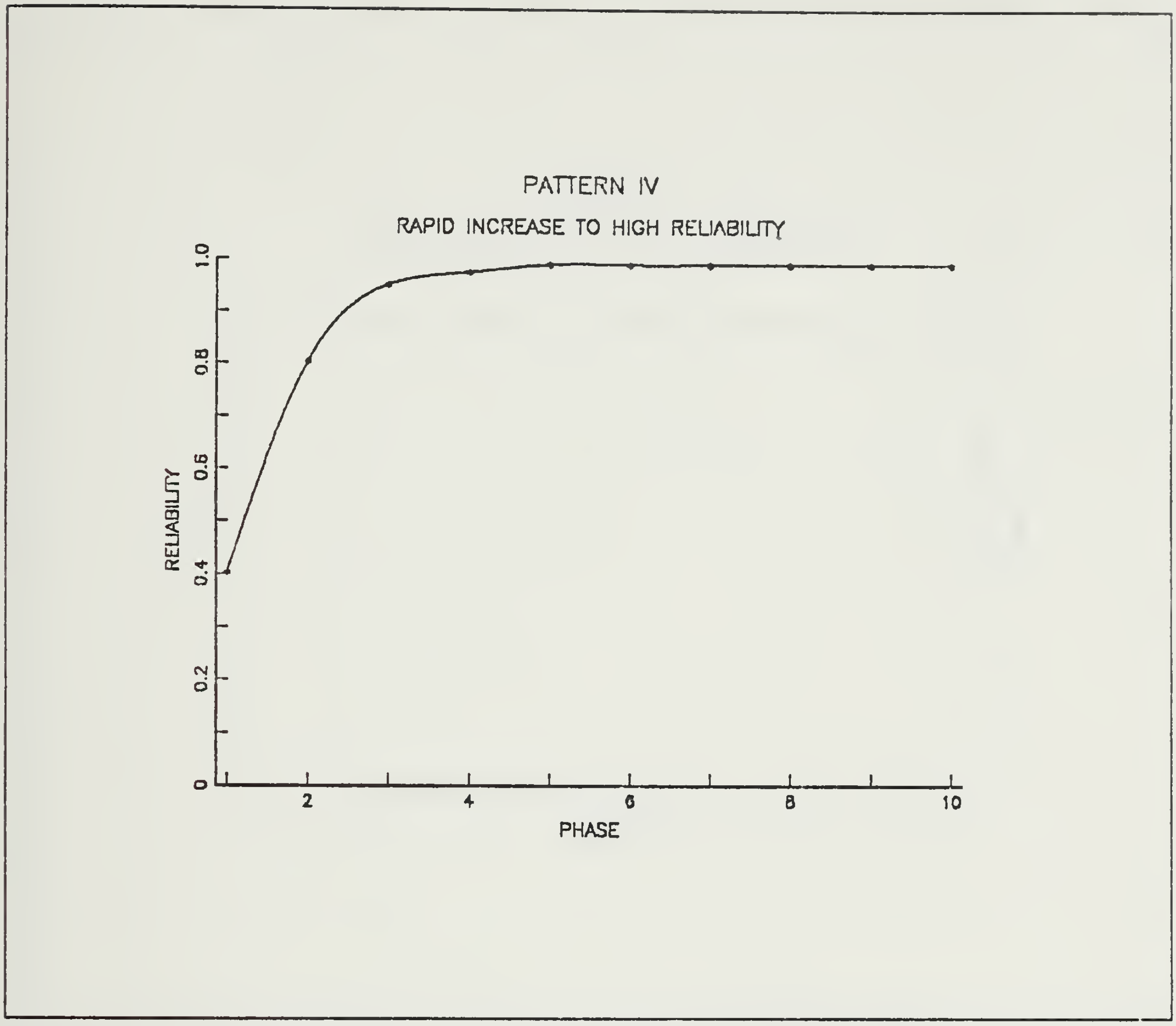

Figure 5. Pattern IV

Table 10. PATTERN IV USER INPUTS

CAUSE	PHASE PROBABILITY OF SUCCESS									
	1	2	3	4	5	6	7	8	9	10
1	.98	.99	.99	.995	.998	.998	.998	.998	.998	.998
2	.95	.98	.99	.995	.998	.998	.998	.998	.998	.998
3	.82	.96	.99	.995	.998	.998	.998	.998	.998	.998
4	.80	.96	.99	.995	.998	.998	.998	.998	.998	.998
5	.66	.90	.99	.995	.998	.998	.998	.998	.998	.998

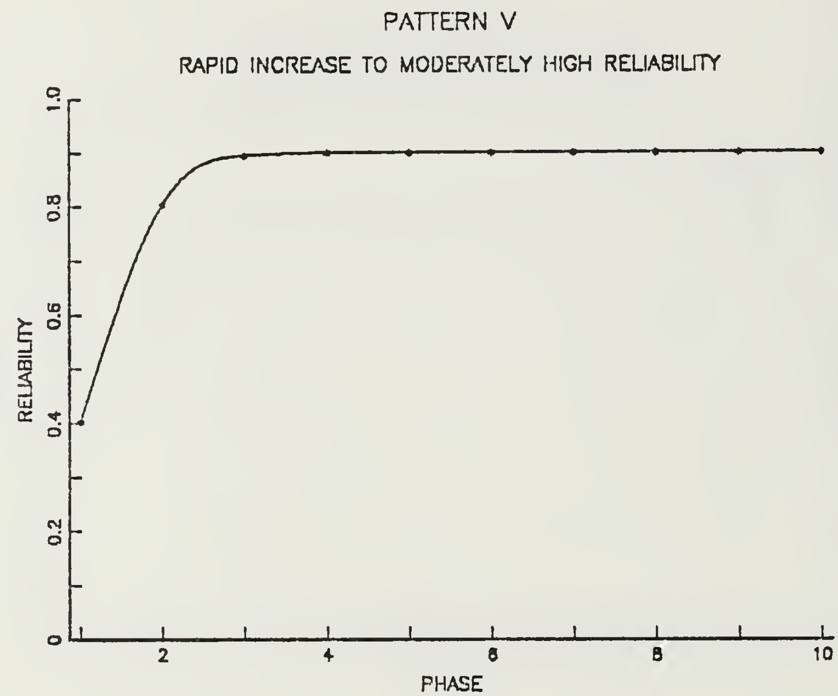

Figure 6. Pattern V

Table 11. PATTERN V USER INPUTS

CAUSE	PHASE PROBABILITY OF SUCCESS									
	1	2	3	4	5	6	7	8	9	10
1	.98	.99	.99	.99	.99	.99	.99	.99	.99	.99
2	.95	.98	.99	.99	.99	.99	.99	.99	.99	.99
3	.82	.96	.98	.98	.98	.98	.98	.98	.98	.98
4	.80	.96	.97	.975	.975	.975	.975	.975	.975	.975
5	.66	.90	.96	.961	.961	.961	.961	.961	.961	.961

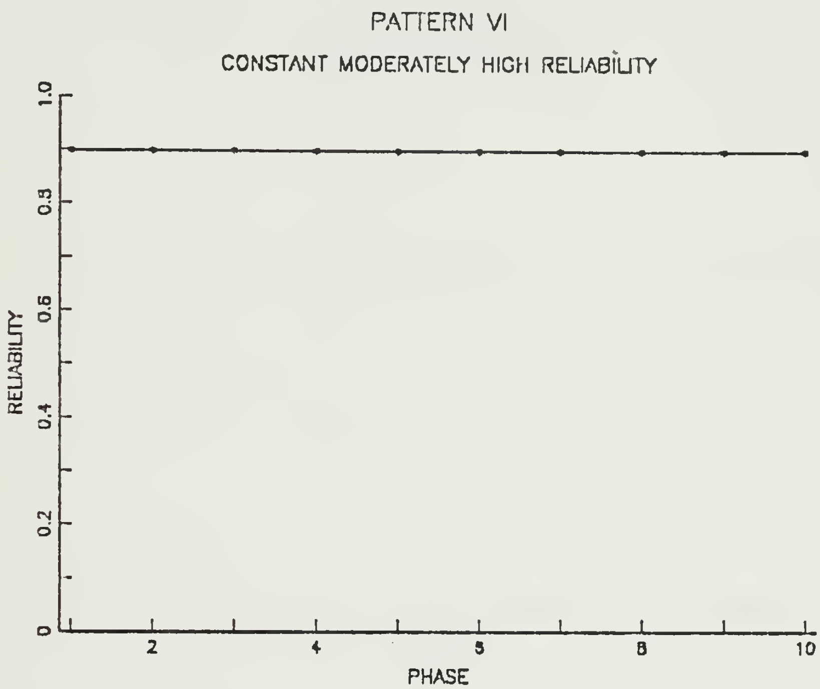

Figure 7. Pattern VI

Table 12. PATTERN VI USER INPUTS

CAUSE	PHASE PROBABILITY OF SUCCESS									
	1	2	3	4	5	6	7	8	9	10
1	.99	.98	.98	.975	.97	.97	.975	.98	.98	.99
2	.98	.98	.975	.97	.99	.99	.97	.975	.98	.98
3	.98	.975	.97	.99	.98	.98	.99	.97	.975	.98
4	.975	.97	.99	.98	.98	.98	.98	.99	.97	.975
5	.97	.99	.98	.98	.975	.975	.98	.98	.99	.97

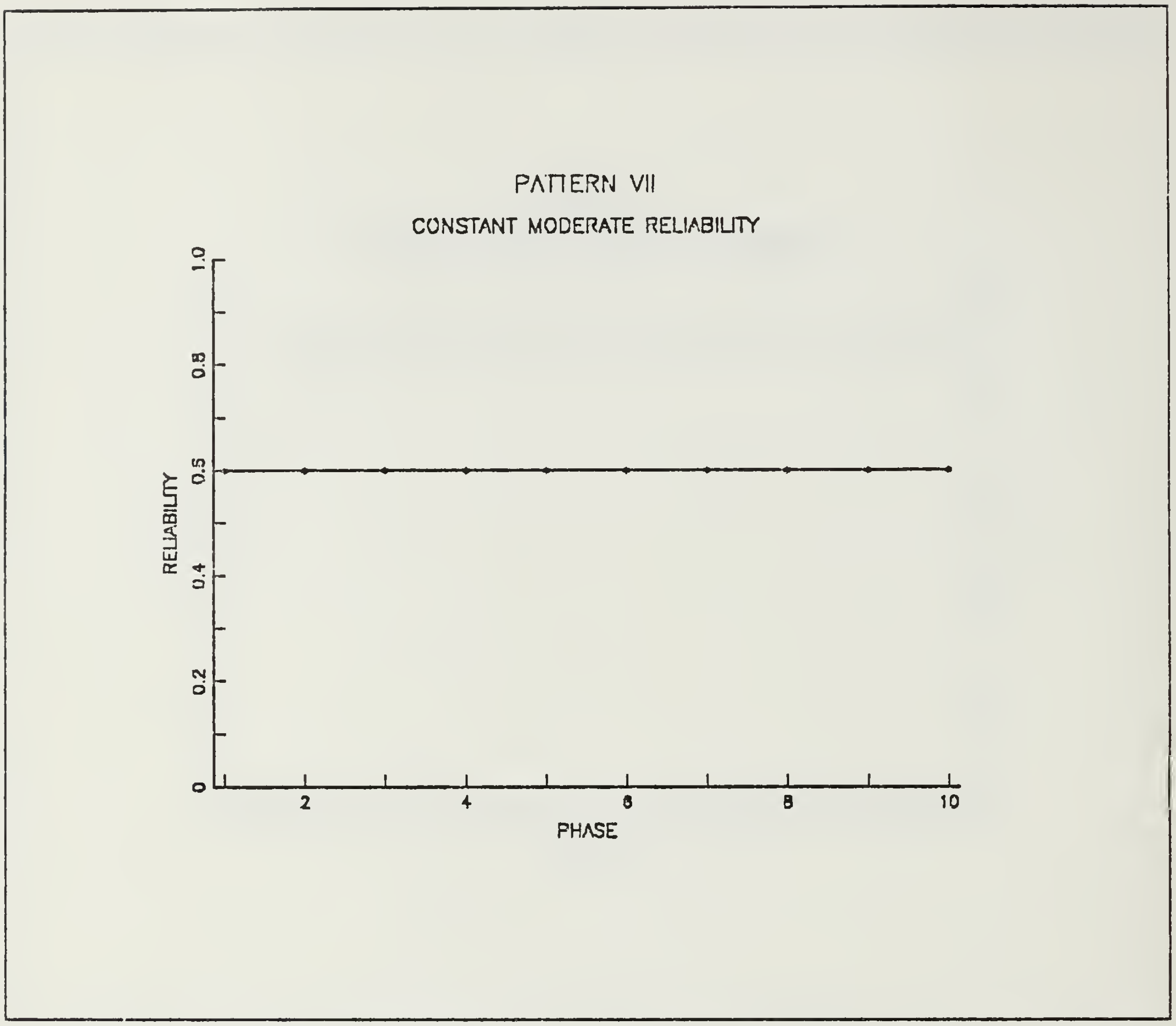

Figure 8. Pattern VII

Table 13. PATTERN VII USER INPUTS

CAUSE	PHASE PROBABILITY OF SUCCESS									
	1	2	3	4	5	6	7	8	9	10
1	.99	.78	.90	.96	.90	.90	.96	.90	.78	.99
2	.78	.90	.96	.90	.99	.99	.90	.96	.90	.78
3	.90	.96	.90	.99	.78	.78	.99	.90	.96	.90
4	.96	.90	.99	.78	.90	.90	.78	.99	.90	.96
5	.90	.99	.78	.90	.96	.96	.90	.78	.99	.90

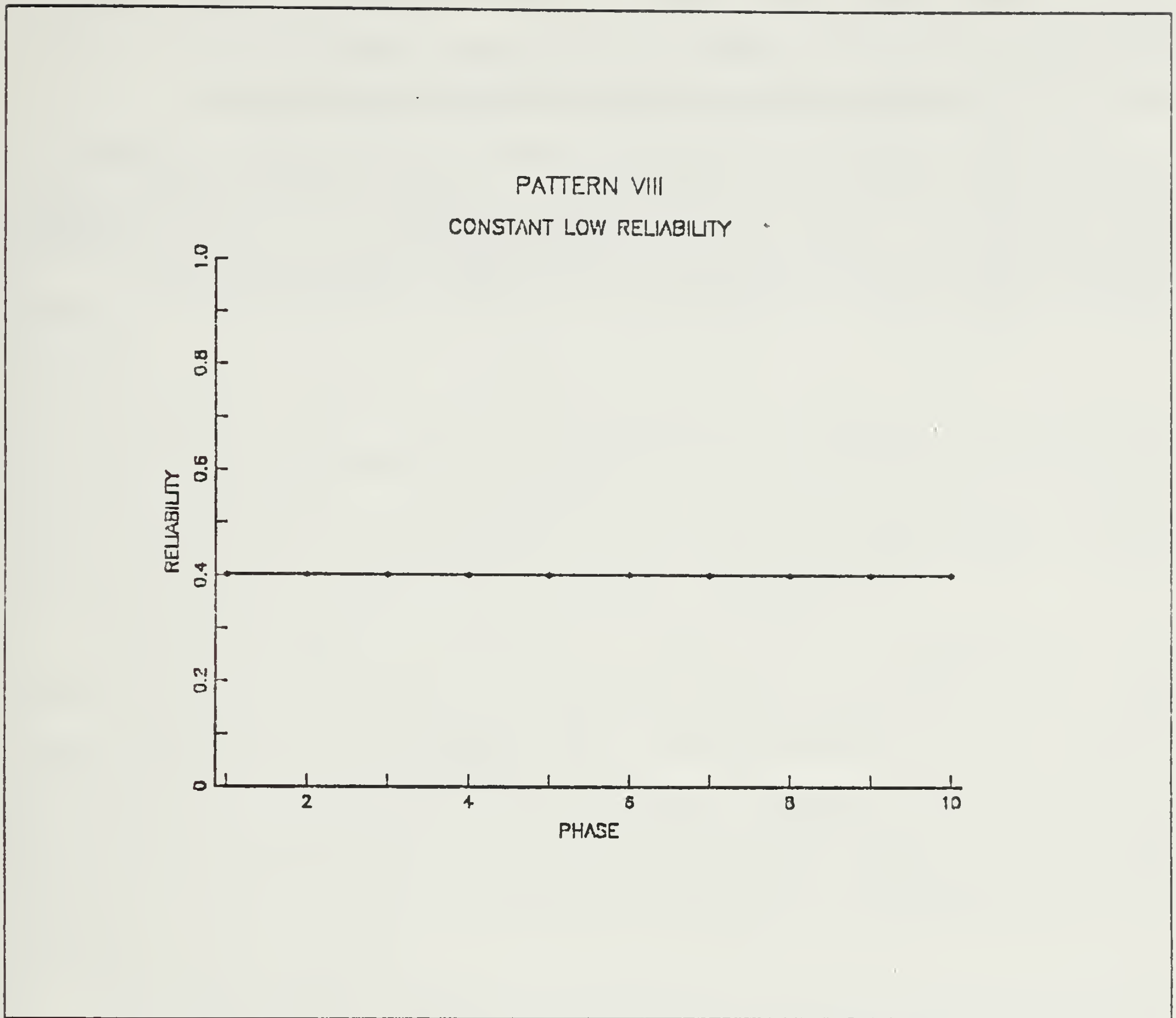

Figure 9. Pattern VIII

Table 14. PATTERN VIII USER INPUTS

CAUSE	PHASE PROBABILITY OF SUCCESS									
	1	2	3	4	5	6	7	8	9	10
1	.98	.95	.82	.80	.66	.66	.80	.82	.95	.98
2	.95	.82	.80	.66	.98	.98	.66	.80	.82	.95
3	.82	.80	.66	.98	.95	.95	.98	.66	.80	.82
4	.80	.66	.98	.95	.82	.82	.95	.98	.66	.80
5	.66	.98	.95	.82	.80	.80	.82	.95	.98	.66

this paper, only considers the number of trials and failures in the current phase. All previous data is discarded since, presumably, the system reliability fluctuates from phase to phase thereby negating the underlying assumption of constant reliability.

Of the eight reliability growth patterns being simulated, only four (including the three cases of constant system reliability, Figures 7, 8, and 9) could have been produced with the simulation as originally written. Therefore, the addition of fixed phase reliability will enable a more rigorous examination of the proposed reliability estimators.

IV. EVALUATION AND INTERPRETATION

This chapter describes results of the simulation runs for each pattern. The performance of the models are reduced to graphical form to facilitate comparison. Only a few of the more representative runs are included in this chapter due to time and space constraints. The graphical results of many of the remaining simulation runs are in Appendix C .

The analysis is organized according to the reliability growth patterns presented in the previous chapter. The performance of each of the two reliability growth models will be evaluated with respect to the growth pattern. The analysis focuses on how well the growth model tracked the actual reliability growth pattern. Included in this evaluation is an examination of the standard deviations of the two estimators.

It is not the purpose of this research to conduct enough simulation runs so that precise rules can be established for the selection of discounting parameters. This would require analyzing many more different types of reliability growth patterns. In addition, the choice of discounting methods and their respective parameters is more a function of the particular system being evaluated than a function of the reliability growth model. Each system will have varying characteristics as will the personnel responsible for conducting the testing procedures.

The reliability growth patterns will be referred to by their respective Roman numeral designations. These numerals are listed on the figures describing each pattern in the previous chapter and are summarized below for completeness:

1. Pattern I - Convexly increasing reliability.
2. Pattern II - Reliability initially increasing then decreases for several phases before resuming its upward trend.
3. Pattern III - Reliability increases rapidly to approximately 0.80. Reliability then remains constant for several phases before increasing to its final level of approximately 0.90.
4. Pattern IV - Reliability increases rapidly to approximately 0.99.
5. Pattern V - Reliability increases rapidly to approximately 0.90.
6. Pattern VI - Reliability starts and remains constant at 0.90.
7. Pattern VII - Reliability starts and remains constant at 0.60.
8. Pattern VIII - Reliability starts and remains constant at 0.40.

A. CONSTANT RELIABILITIES -- PATTERNS VI, VII, AND VIII

1. Standard Discount Method

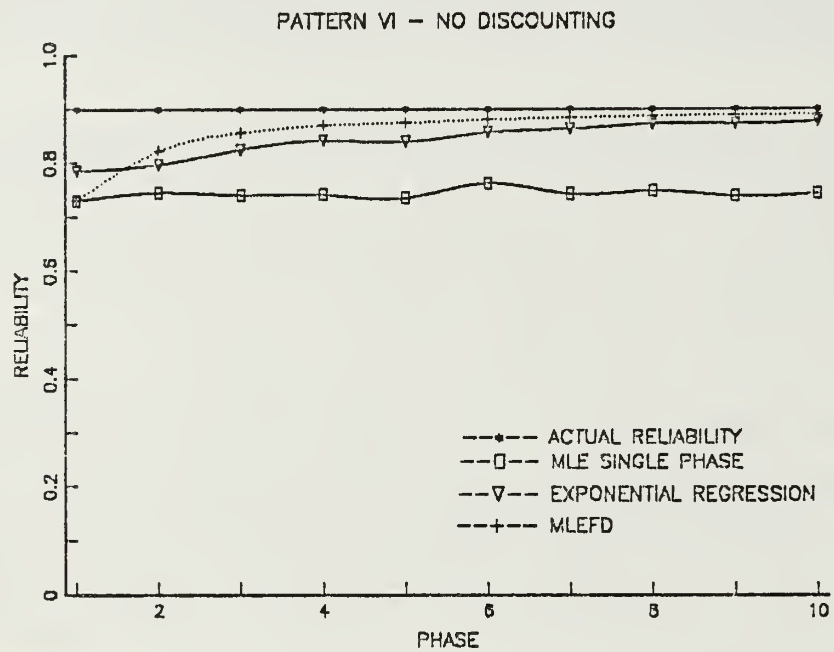

Figure 10. Pattern VI. No Discounting

Figure 10 shows the performance of the two reliability growth models with respect to Pattern VI. In this particular figure the discount fraction, F, is set at zero (previous system failures are not discounted). As one might expect, both models performed well. Since the actual system reliability is constant across all phases, the basic underlying assumption of constant reliability for the maximum likelihood estimate is valid. Therefore, previous data is compatible with current data without the use of failure discounting techniques. Thus, including all previous failure data such as done in the Maximum Likelihood Estimate with Failure Discounting (MLEFD) will lead to a more accurate estimation of reliability. It is also interesting to note that the single phase maximum likelihood estimate discussed briefly in the previous chapter resulted in an estimate of 0.743 for phase ten. The single phase maximum likelihood estimate considers only the number of trials and failures in the current phase and ignores past test data. The expected value of this estimate, using Equation 2.5 is 0.744, thus supporting the validity of the simulation process.

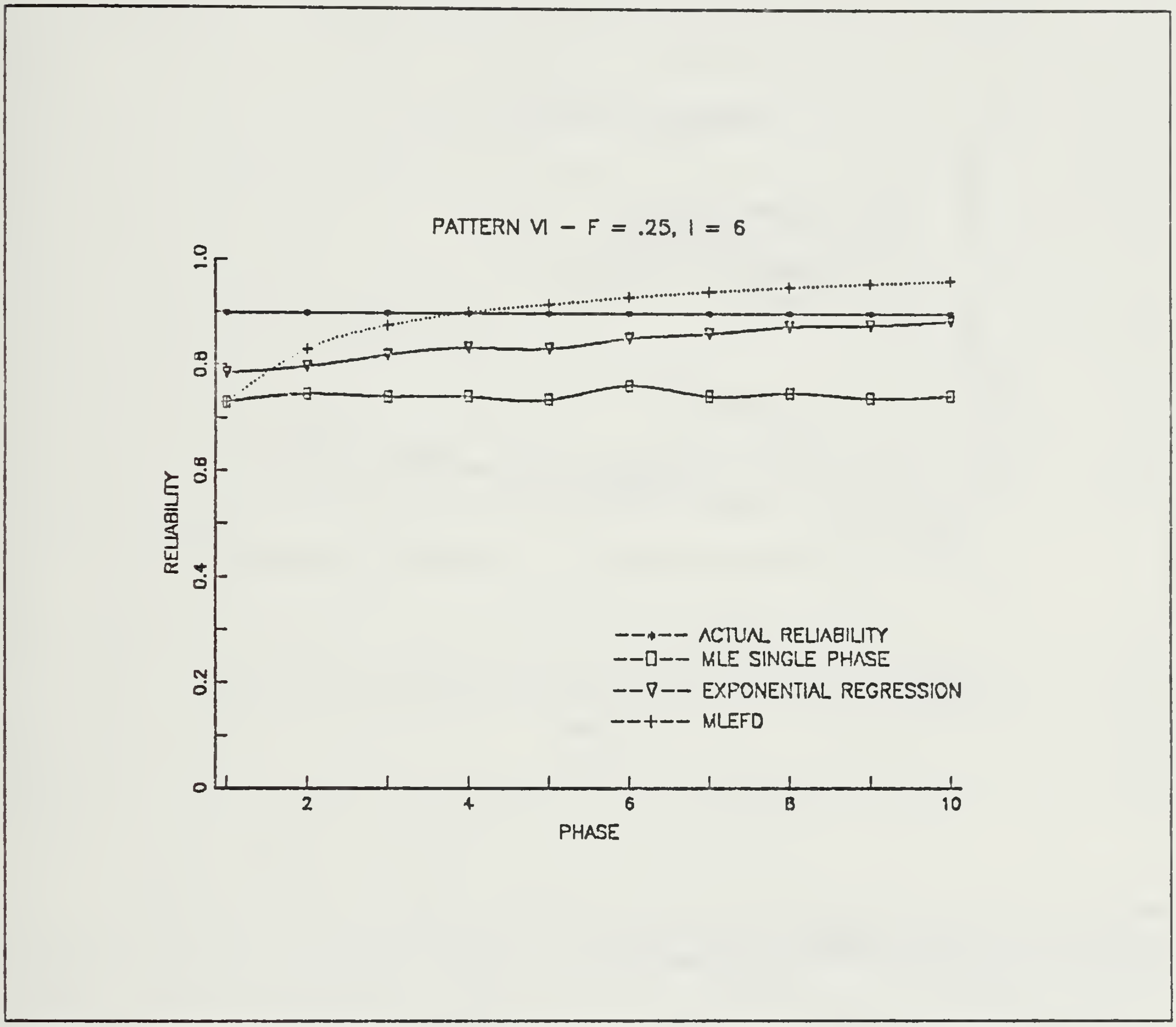

Figure 11. Pattern VI, F = .25 and I = 6

Both models did very well in terms of the stability of the estimate when the actual system reliability was high. This is largely due to the fact that high system reliability implies that more test trials will be required to produce a failure. More test trials translates into additional data for the two models and, therefore, a less variable estimate. It should be noted, however, that when actual system reliability is constant, the MLEFD does better than the exponential regression model. The standard deviation of the

MLEFD is less than half that of the exponential regression estimate. Appendix C also contains the graphs of the standard deviations of the two estimates for all eight patterns for the interested reader.

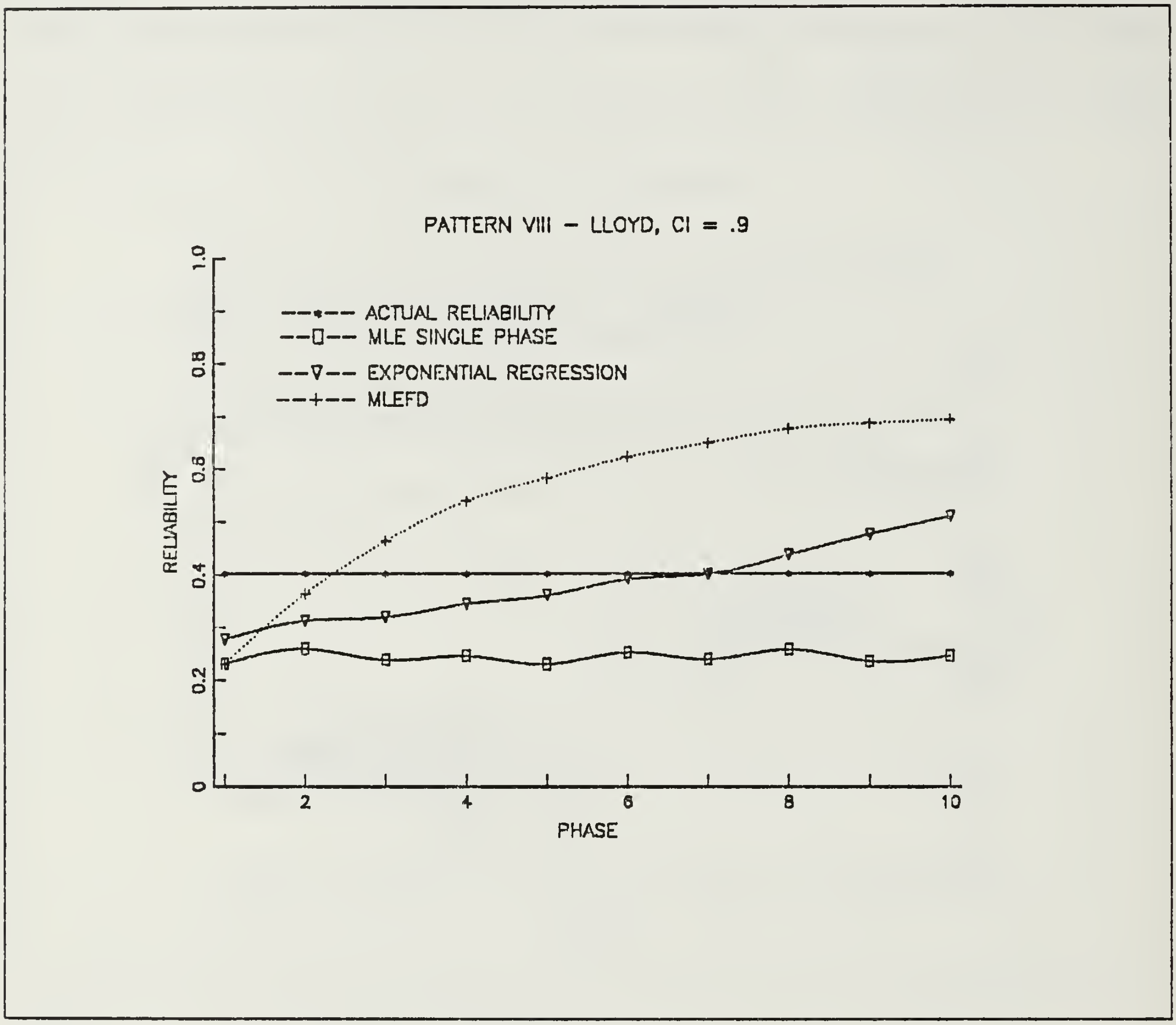

Figure 12. Pattern VIII, Lloyd method - CI = .9

The standard discount method will lead to an overestimation of reliability when applied to the MLEFD for the constant system reliability patterns. Figure 11 depicts the results of applying this method with the parameter values set at F = 0.25 and I = 6. While only slightly affecting the exponential regression model, discounting previous failures in this manner resulted in the MLEFD overestimating system reliability fairly significantly after phase four. This is not unexpected since, as stated before, the assumption of constant system reliability is valid in this instance. It is important to note, however, the sensitivity of the MLEFD model to discount procedures compared to the exponential regression model. Despite using a relatively small value for F the predicted reliabilities were markedly altered. This characteristic of the MLEFD is also evident in the patterns to follow.

2. Lloyd Discount Method

The Lloyd discount failure discounting method has only one parameter, the confidence interval. For the purposes of this study the only values for CI that were addressed were 0.80 and 0.90. The formula for applying the Lloyd discount method, Equation 2.2, is constructed so that the higher the value of CI, the less a previous failure is discounted. Figure 12 graphically represents Pattern VIII and the results of applying the Lloyd method with CI = 0.90 to the two models. Once again, the effect on the MLEFD is rather dramatic with a phase ten estimate of reliability of approximately 0.70 while the actual system reliability is 0.40. The effect on the exponential regression model, although significant after phase seven, is not nearly so severe. This marked effect of applying the Lloyd discount method to the two reliability growth models continued with the other patterns as will be seen. Again, the important thing to note is the extreme sensitivity of the MLEFD to failure discounting in general and to the chosen values of the discount parameters in particular.

B. RAPID RELIABILITY GROWTH -- PATTERNS IV AND V

The discussion of the results achieved for these patterns will center around Pattern V. Although the tendencies of the two models were the same for both patterns, the graphical portrayal of identified trends is better observed with this pattern than with Pattern IV where the final system reliability approaches 0.99.

1. Standard Discount Method

Figure 13 depicts Pattern V with the discount fraction, F, set at zero. Both the MLEFD and the exponential regression model perform well. It is significant to note, however, that the exponential regression model converged to the actual system reliability much faster than did the MLEFD. This is due in large part to the use of linear regression techniques in this model. The standard deviation of the exponential regression was comparable to the MLEFD for this pattern although still slightly larger. Both the MLEFD and the exponential regression model outperformed the single phase maximum likelihood estimate thus lending validity to the use of reliability growth models to estimate system reliability.

Figure 14 shows Pattern V with F = 0.50 and I = 3 trials. This high a rate of failure discounting caused the MLEFD to yield a phase ten estimate of 0.997 when the

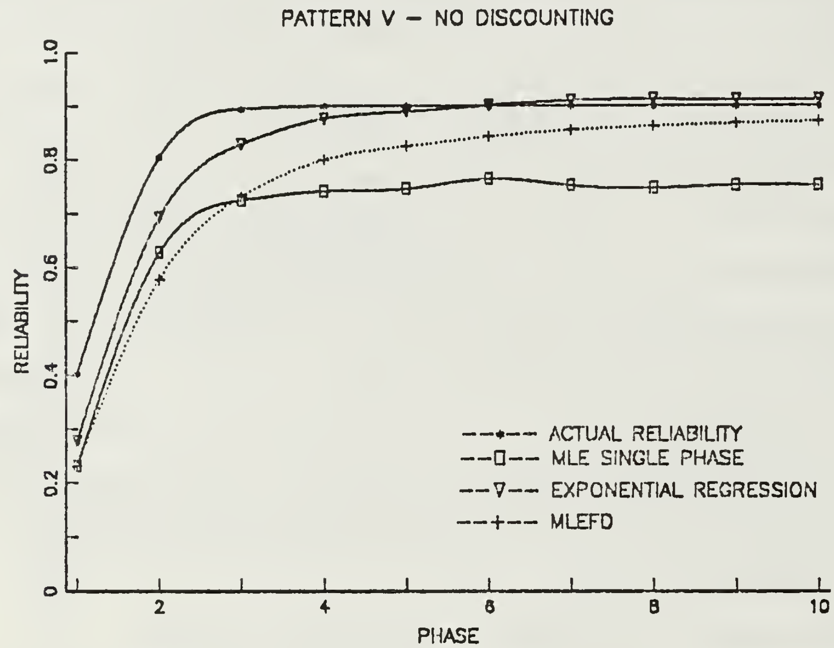

Figure 13. Pattern V, No Discounting

actual reliability was 0.900. A correlate of this high estimate of reliability is that the standard deviation associated with the MLEFD became very small relatively quickly. The sensitivity of the MLEFD to failure discounting is again apparent in this figure.

The exponential regression model, on the other hand, does remarkably well. It quickly approached actual system reliability. However, the standard deviation associated with this estimate remained high at a relatively constant value of 0.20. So, while the mean estimate of system reliability was very good, the associated variance of the estimate is greater than one would desire.

The MLEFD performed quite well when F was set at 0.25 and I was put at 15 as shown in Figure 15. The exponential regression model did slightly overestimate the

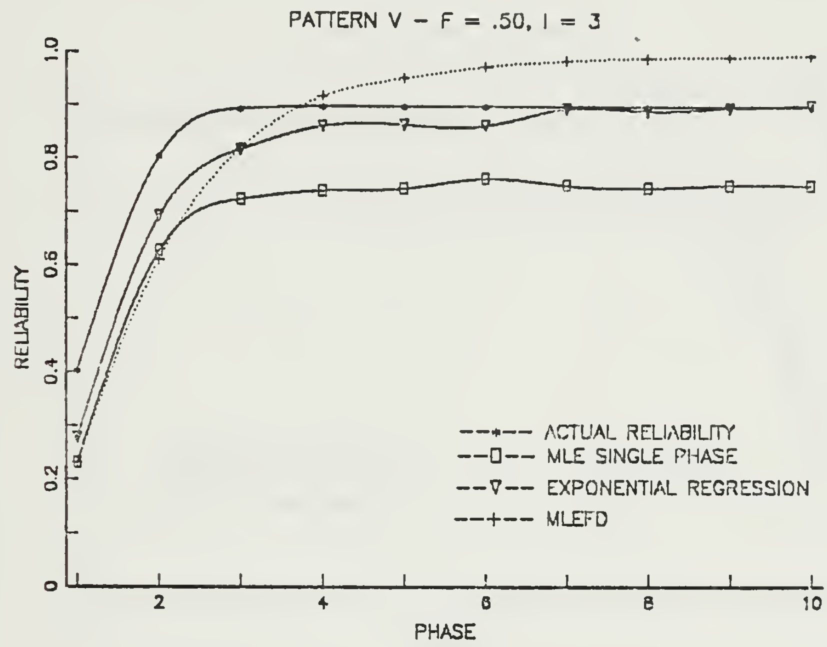

Figure 14. Pattern V, F = .5 and I = 3

system reliability after the sixth phase however, it was more proficient at matching the actual growth of reliability in the system. There was not a significant difference in the standard deviations of the two estimates.

2. Lloyd Discount Method.

Figure 16 depicts Pattern V with a confidence interval value of 0.80. Note that, as on the constant system reliability patterns discussed previously, application of the Lloyd discount method in its derived form caused both estimators to predict system reliabilities that were markedly greater than the actual system reliability. This trend was consistent no matter what values of CI were used.

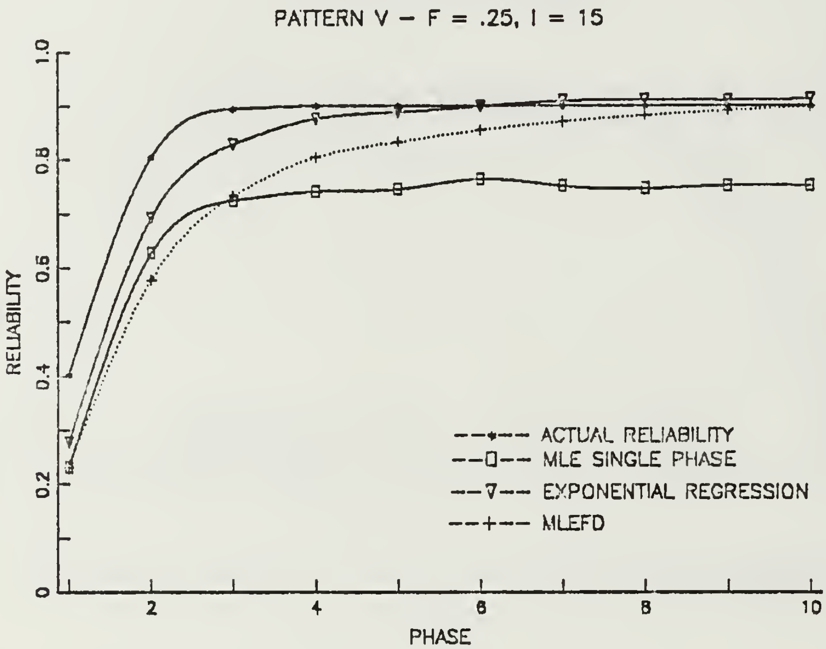

Figure 15. Pattern V, F = .25 and I = 15

The conventional estimate of reliability, the single phase maximum likelihood estimate, is not affected by discount procedures since only current test data is considered. The discounting methods addressed in this paper are only applied to failures occurring in previous phases.

C. DECREASING RELIABILITY - PATTERN II

1. Standard Discount Method

Figure 17 shows Pattern II with no failure discounting applied. Of special interest in this graph is the performance of the two reliability growth models during the period of declining reliability. The exponential regression model was the only model that actually demonstrated a decrease in reliability although it still overestimated the actual

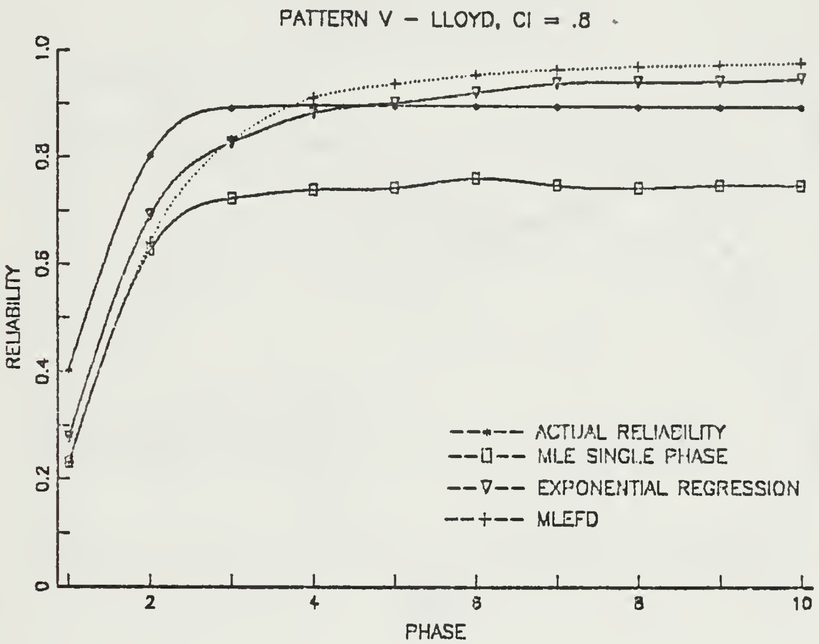

Figure 16. Pattern V, Lloyd method, CI = .8

reliability during this period. The MLEFD, although slowing in terms of growth rate, failed to capture the trend of the actual reliability. The lack of ability to successfully predict the downward trend in actual reliability on the part of the MLEFD was present regardless of what values were assigned the discount parameters. In fact, as one might expect, discounting previous failures exacerbated the problem.

Figure 18 shows Pattern II with F = 0.75 and I = 6. The exponential regression model did very well, particularly during the phases where system reliability was declining. The MLEFD, in contrast, performed poorly overestimating system reliability significantly after phase three and failing to indicate that system reliability decreased.

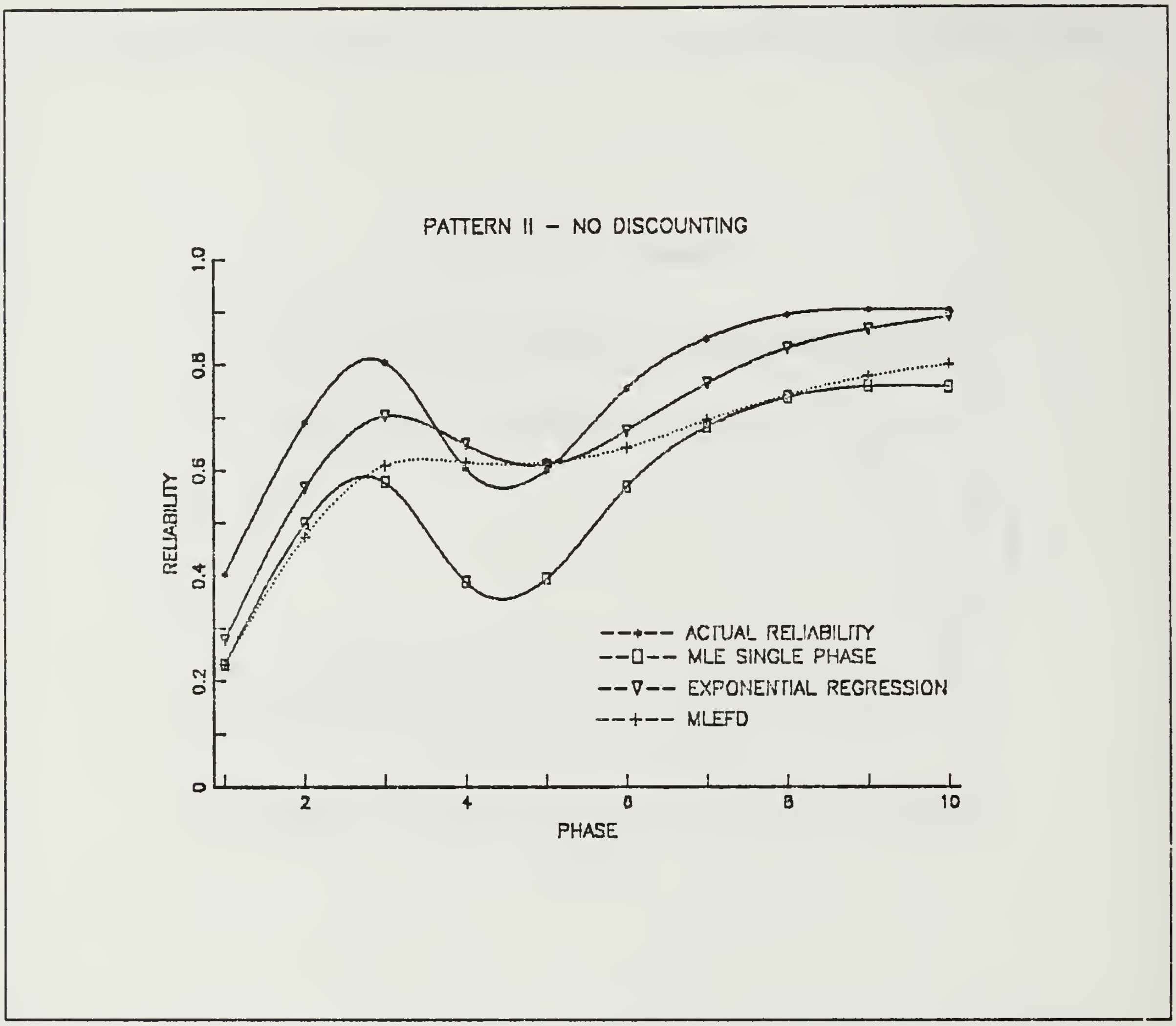

Figure 17. Pattern II, No Discounting

The standard deviation of the exponential regression estimate still exceeded that of the MLEFD. This characteristic is fairly consistent for all the patterns that were examined in the course of evaluating these two models. It should be noted, however, that a small standard deviation does little good if the estimator is yielding markedly incorrect predictions.

2. Lloyd Discount Method

Figure 19 shows Pattern II with CI = 0.90. The application of the Lloyd method profoundly affected the performance of the MLEFD while only slightly altering the exponential regression model. This method of failure discounting did cause both models to overestimate system reliability by phase nine but, again, had less of an effect

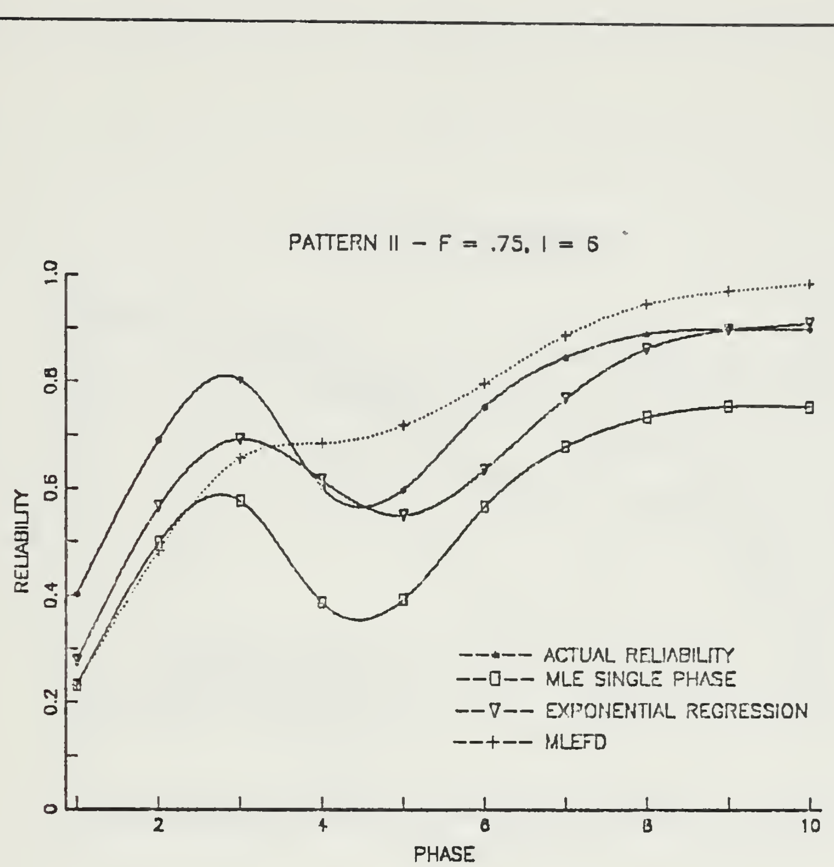

Figure 18. Pattern II, F = .75, I = 6

on the exponential regression model. It is especially noteworthy to observe the effect of the Lloyd method on the MLEFD during the period of decreasing system reliability. The real sensitivity of the MLEFD to discount methods and parameters is particularly evident in this pattern.

D. INTERMITTANT RELIABILITY GROWTH - PATTERN III

1. Standard Discount Method

Figure 20 depicts the performance of the two models when no failure discounting is applied. The exponential regression model outperformed the MLEFD at every phase. Again, the exponential regression model succeeded in capturing the trend of

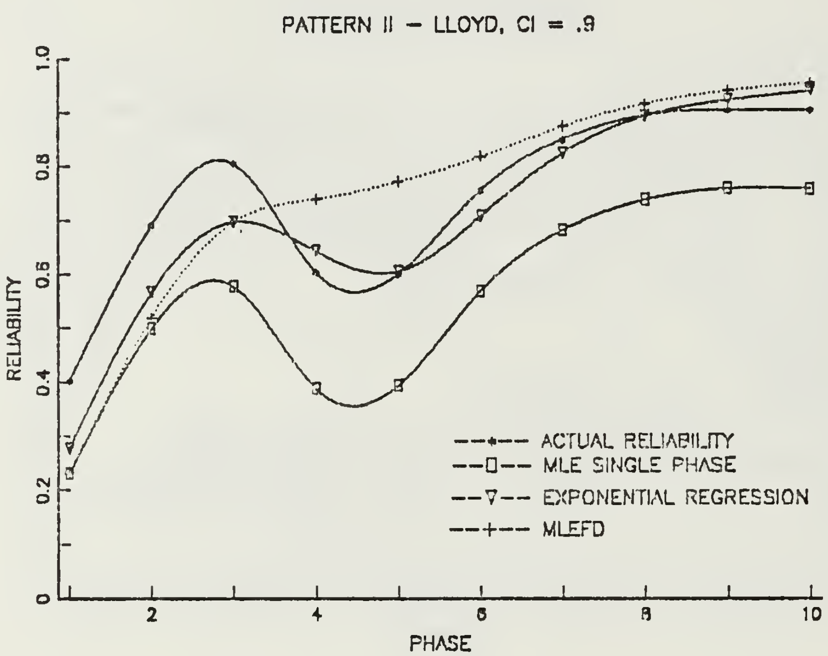

Figure 19. Pattern II, Lloyd method, CI = .9

the reliability growth pattern from phase four to phase six whereas the MLEFD exhibited a constant rate of increase.

When the discount parameters F and I were set at 0.25 and six respectively, the MLEFD did much better as can be seen in Figure 21. Both models converged to the correct prediction of system reliability by phase ten. However, the exponential regression model converged to the correct value much more quickly than did the MLEFD. The standard deviations of the two estimates were not too significantly different from each other for this particular pattern and discount method.

The performance of the two models on reliability growth patterns II and III demonstrate one of the inherent advantages of the exponential regression methodology. Since this method makes use of the techniques of linear regression, it is better able to

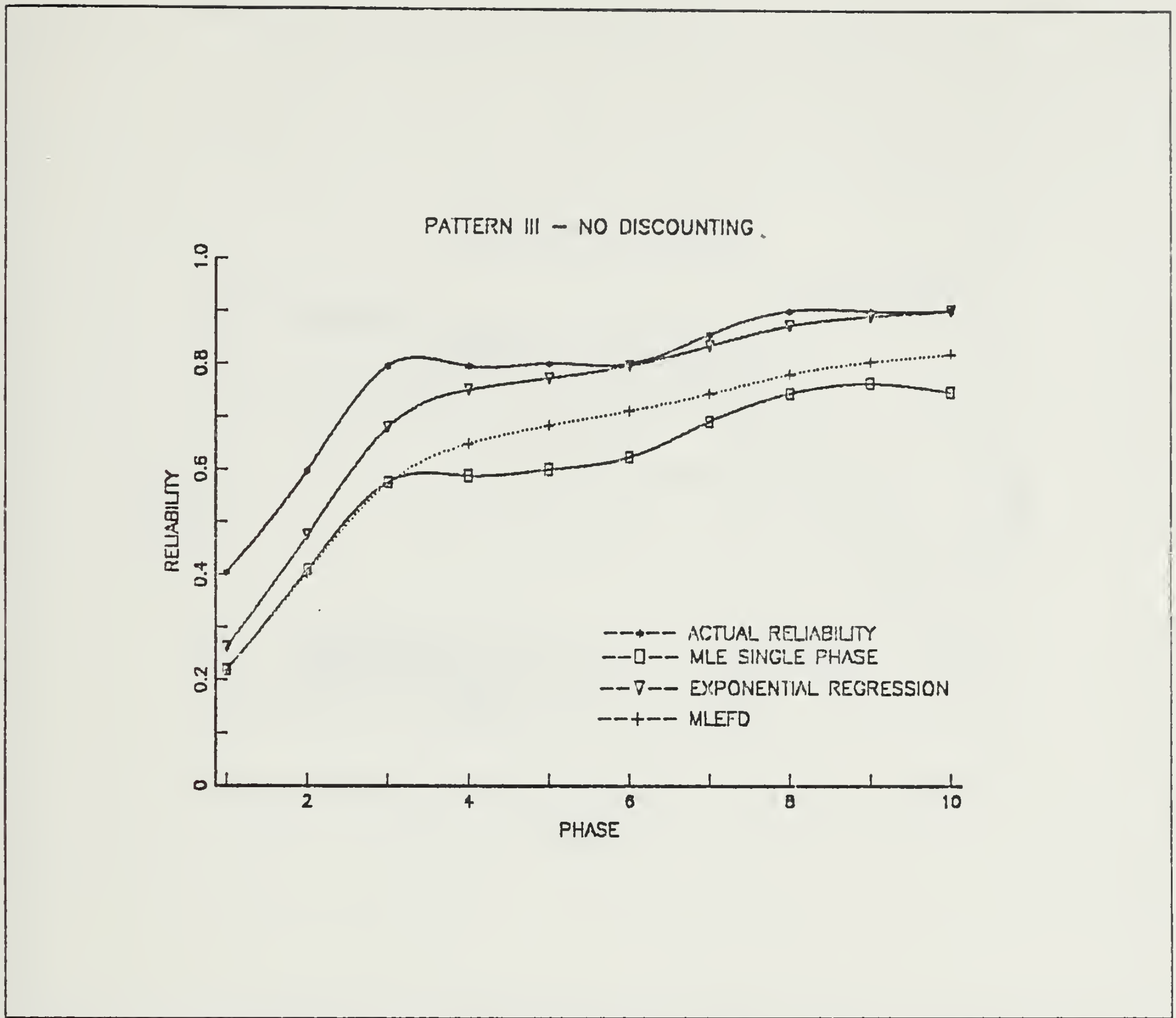

Figure 20. Pattern III, No Discounting

track changing system reliability than is the MLEFD. This attribute of the exponential regression model will be developed further in the next chapter.

2. Lloyd Discount Method

Figure 22 shows how the models performed using Lloyd failure discounting with CI = 0.90. Again, applying the Lloyd method caused the MLEFD to overestimate system reliability after phase four. Some slight overestimation did occur in the exponential regression model during phases nine and ten however, the overall effect was to enable the model to better track actual system reliability during the middle phases. During the discussion of the discounting methodologies in Chapter II, it was noted that the Lloyd failure discount method is applied after every trial that did not result in the reoccurrence of a previous failure cause. This means of application is a likely source of

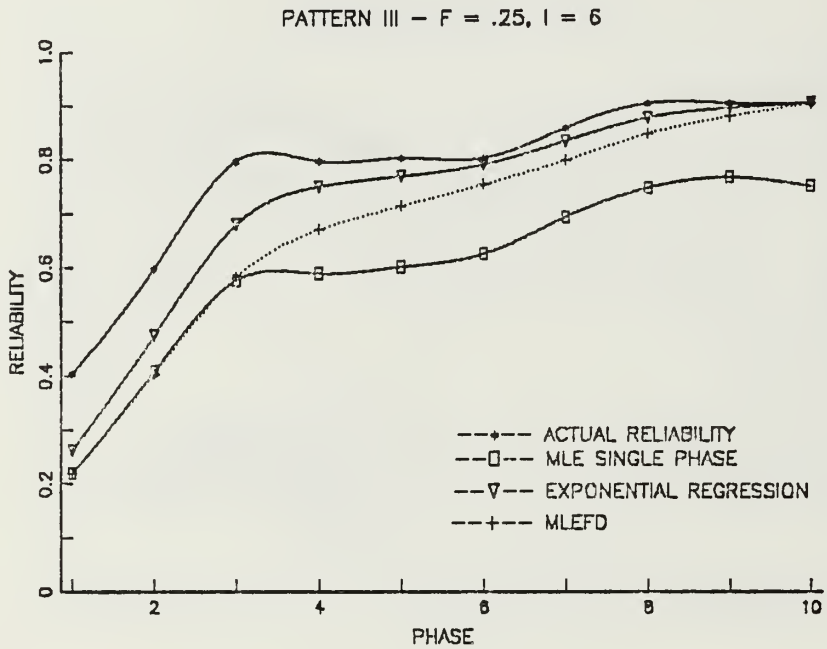

Figure 21. Pattern III, F = .25, I = 6

the overestimation characteristics of both models when Lloyd discounting is applied. An idea on a possible remedy to this problem will be presented at the end of this chapter.

E. CONVEX RELIABILITY GROWTH - PATTERN I

Most reliability growth patterns are thought to be essentially concave in shape, much like Patterns IV and V. This is normally a valid assumption in that major improvements in the reliability of a new system most often occur during the early stages of development as the obvious flaws in the design and manufacture become apparent. It is not inconceivable, however, to experience a period of convex growth such as shown in Pattern I. One would want to have confidence in the chosen reliability estimator to correctly estimate system reliability regardless of its actual form.

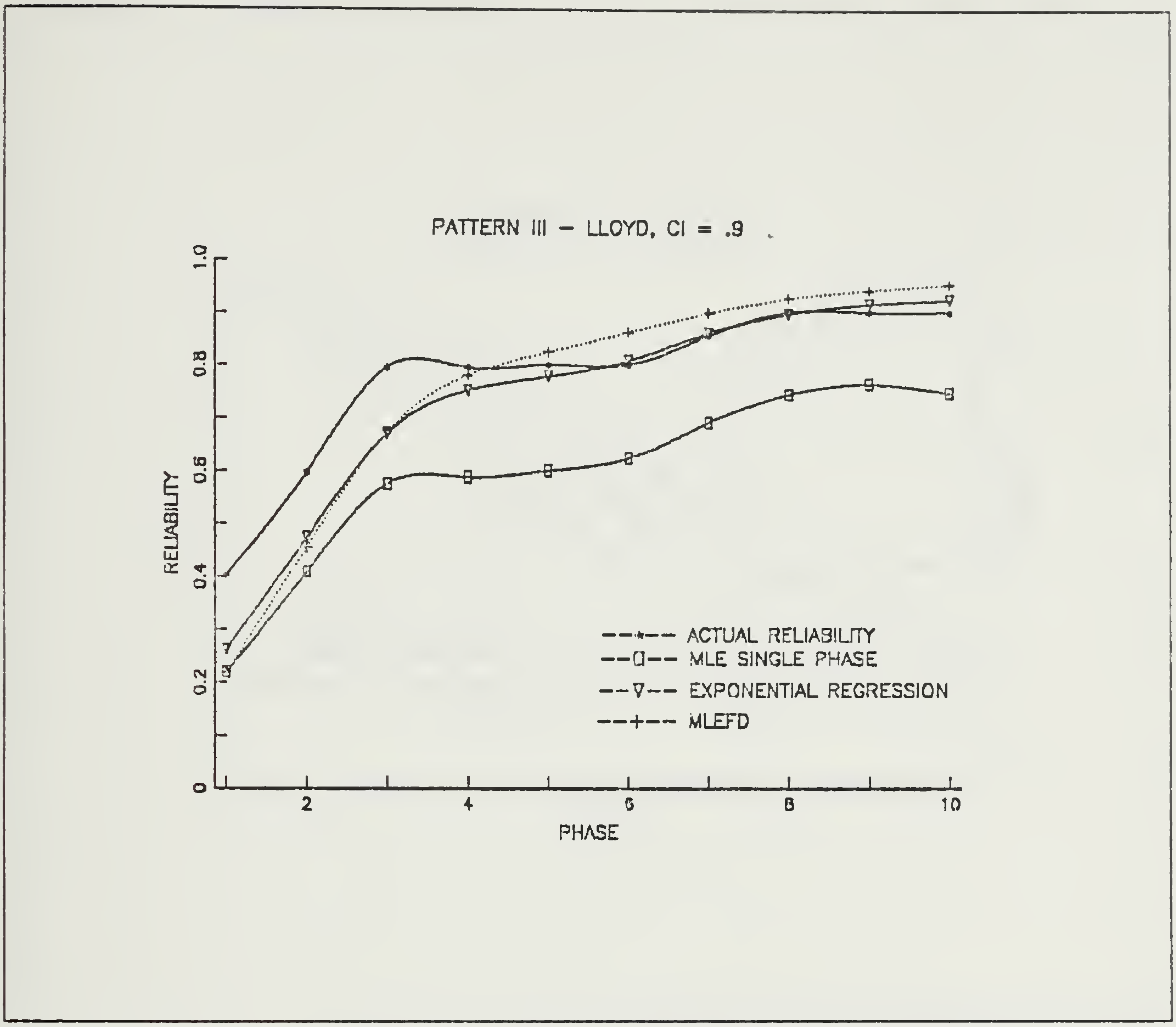

Figure 22. Pattern III, Lloyd method, CI = .9

1. Standard Discount Method

Figure 23 shows the performance of the two models with no failure discounting being applied. It is apparent that, even without the benefit of failure discounting techniques, the exponential regression model does well. It is able to reflect the trend of actual system reliability growth and the actual estimates are fairly close to the true system reliability. The MLEFD does not do so well and, in fact, is even outperformed by the single phase maximum likelihood estimate in the later phases. One would expect in this case to have significant results when failure discounting is applied due to the demonstrated sensitivity of the MLEFD to this procedure.

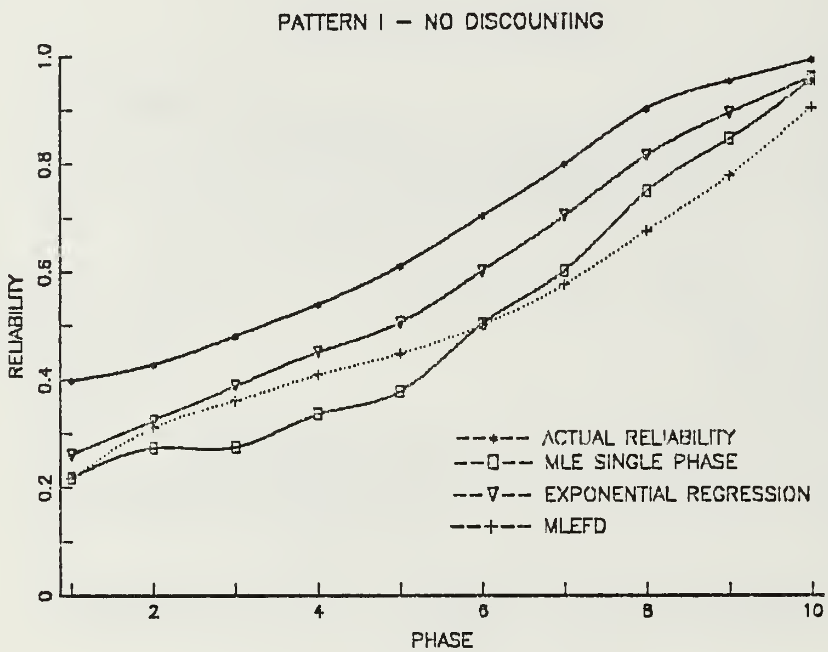

Figure 23. Pattern I, No Discounting

Figure 24 depicts Pattern I with F = .75 and I = 6. Although failing to capture the shape of the actual reliability pattern, the phase estimates produced by the MLEFD are quite good. Fractionally removing 75 percent of a previous failure after six successful trials is a substantial discount and one would expect that such a large discounting scheme would lend itself more to the conventional concave reliability growth patterns. This particular type of scenario would benefit greatly by the ability to alter the discount parameters during the course of the test. Initially, one would not want to discount previous failures by a large amount since actual system reliability is only improving marginally. As the system evolves and reliability begins to make substantial jumps then previous failures should be discounted more. Of course, this presumes a knowledge of

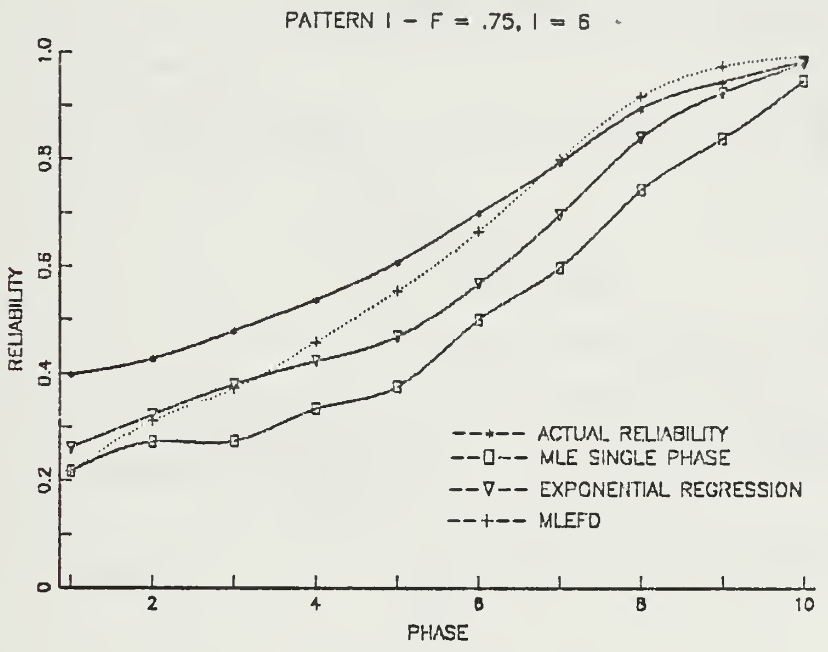

Figure 24. Pattern I, F = .75 and I = 6

the actual system reliability in order to properly design such a plan and this is never the case. However, in depth knowledge of the proposed system could indicate the expected shape of the reliability pattern and this could be made use of in test design. Although changing the discount parameters during the course of the test is possible in actual practice it is not, unfortunately, possible to do with the simulation in its current form. The parameters for the discount procedures remain constant once they are input at the beginning of the simulation.

2. Lloyd Discount Method

Figure 25 shows the Lloyd method with CI = 0.90. The exponential regression model maintains the shape of the true system reliability while converging to the actual

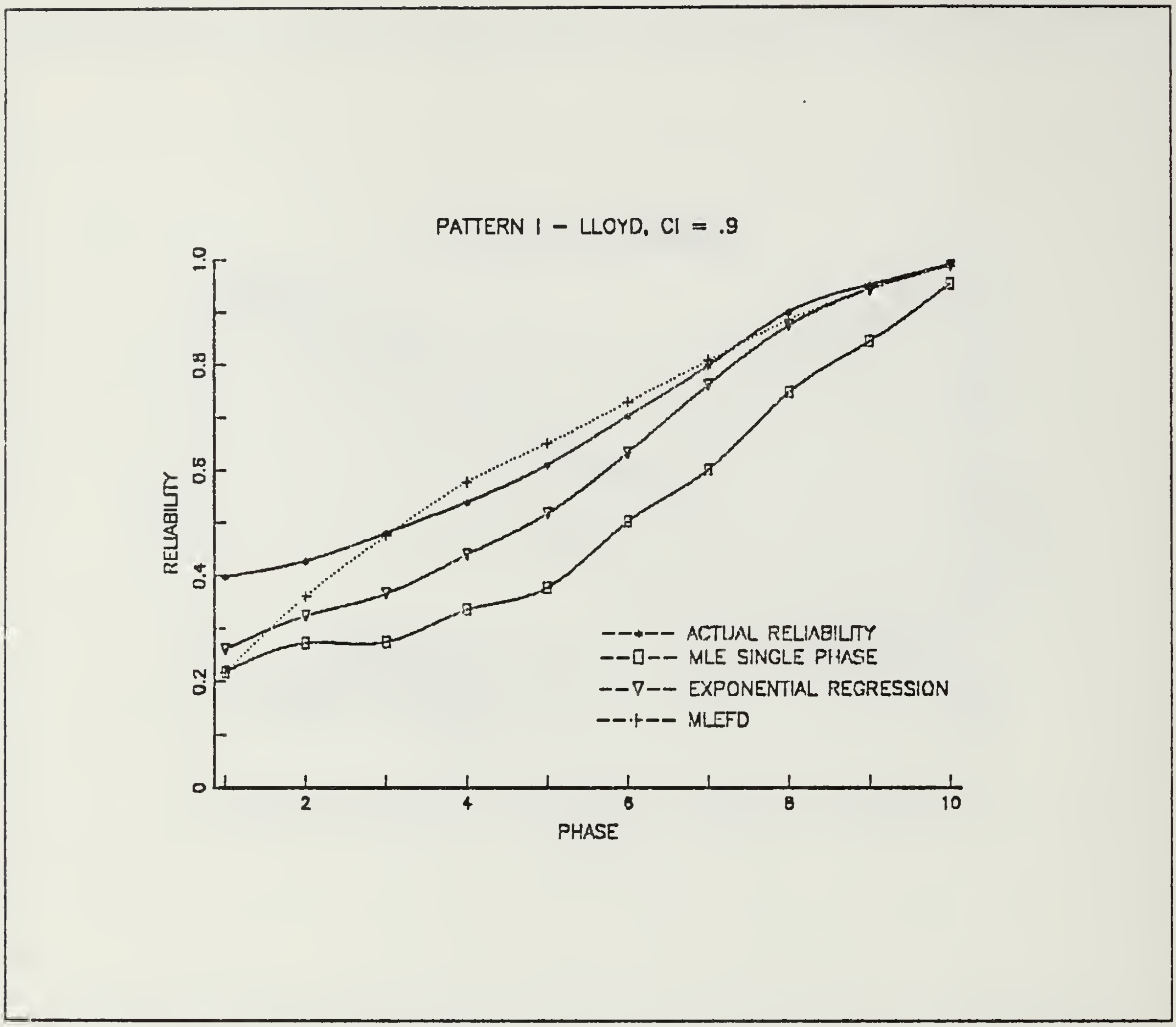

Figure 25. Pattern I, Lloyd method, CI = .9

reliability by phase eight. The MLEFD becomes concave in shape much as it did for the standard discount method shown in Figure 24. However, the intermediate phase reliability estimates are very good. The standard deviation of the MLEFD continues to be less than that of the exponential regression model although, as shown in Chapter II, the mean square error for the MLEFD exceeds that of the exponential regression model.

The Lloyd method does better with this pattern than with any other. Previously, application of this method caused both models to overestimate system reliability quite significantly. In this case, using this method improved the performance of the exponential regression model and, although distorting the shape, resulted in improved reliability estimates from the MLEFD as well.

F. LLOYD FAILURE DISCOUNTING (REVISITED)

Applying the Lloyd failure discounting method in its current form consistently caused both models to overestimate the true system reliability. This was true for the fixed phase reliability growth patterns that were addressed in this paper as well as for the growth patterns that were analyzed in Captain Drake's work previous to this [Ref. 1: pp. 64-65]. One potential source of this problem is that the Lloyd method is applied after every successful trial.

The formula for computing adjusted failures using the Lloyd method is given by Equation 2.2 repeated below:

$$ADJUSTED\ FAILURE = 1 - (1 - CI)^{\frac{1}{T}} \quad if\ T > 0. \tag{2.2}$$

If T equals zero then the adjusted failure value is set to one. This formula is used to compute adjusted failures after each trial in which the cause for a previous failure did not reoccur. Since the problem appears to lie in the number of times the method is applied the inclusion of an interval may lead to more accurate results.

A modified version of the Lloyd discount method which employs an interval of application is as follows:

$$ADJUSTED\ FAILURE = 1 - (1 - CI)^{\frac{1}{M}} \quad if\ M > 0 \tag{4.1}$$

where, in this case,

$$M = INT\left(\frac{T}{LDI}\right)$$

LDI is defined as the Lloyd Discount Interval and becomes another parameter that can be altered by the user of the simulation. If M equals zero then the adjusted failure value is set at one as before. By its construction, one can observe that for all values of T less than the predesignated LDI, then the value of M will be zero. M will increase by one for each group of successful trials meeting the LDI. Note also that specifying an LDI of one allows the user of the simulation to employ the Lloyd method in its original form.

Some preliminary runs of the simulation were made with this modification included. The results were encouraging. Figure 12 examined previously illustrated the effect of the Lloyd method on the two models when system reliability was actually a constant 0.40. Figure 26 shows the result of applying this modified version of the Lloyd method with the value for CI remaining at 0.90 and the LDI set at eight. Both the MLEFD and the

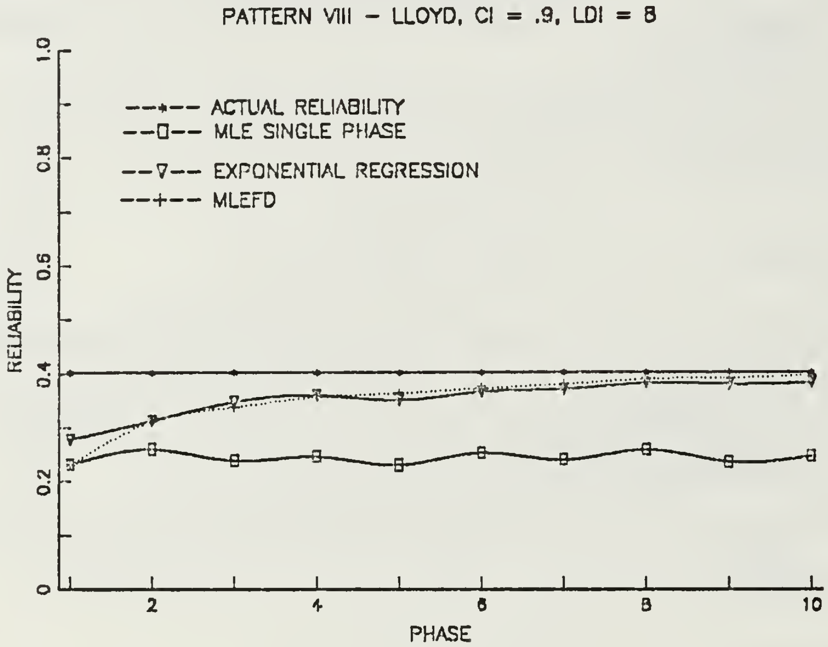

Figure 26. Pattern VIII, Modified Lloyd Method, CI = .9, LDI = 8

exponential regression model performed much better than before. The LDI value of eight resulted in similar improvements for the other constant system reliability patterns.

The results on the other patterns were similar to those represented in Figure 17 but the value for the LDI had to be adjusted for each pattern. Before any specific conclusions can be drawn about the benefits of this modification many more computer runs and more in depth analysis is required. However, based on these preliminary results, the inclusion of an interval does appear to have some merit.

V. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

A. SUMMARY

The purpose of this paper was to evaluate two discrete reliability growth models. Each of these models has the ability to use fractionally discounted failure data to make an estimate of the actual system reliability at any point during the conduct of a test. Fractionally discounting failures occurring early in a test procedure allows the data accumulated during the initial stages of a test to be used in current reliability estimates. Therefore, if the discounting procedure is performed wisely, more accurate reliability estimates with a higher degree of confidence should result.

Two procedures for discounting previous system failures were described in Chapter II. The standard discount method requires the user to specify two parameters, the fraction of failure to be removed and the discount interval. The fraction of failure to be removed, referred to as the discount fraction or F, specifies how much a previous failure is reduced upon each application of the discount procedure. The discount interval, I, is simply the number of successful trials (trials without a reoccurrence of the particular failure cause) that must occur between applications of the procedure. The other failure discounting method was proposed by David Lloyd and is referred to as the Lloyd discount method. This method utilizes only one input parameter, the level of confidence for an upper confidence interval on reliability. This method, in its original form, is applied after every successful trial. A modified Lloyd discounting method which employs an interval of application was derived in Chapter IV.

Two discrete reliability growth models were described. The first model is a form of the maximum likelihood estimate with the addition of failure discounting. Since the maximum likelihood estimate assumes a constant reliability between phases, the addition of failure discounting will enable this model to estimate changing system reliability. The Maximum Likelihood Estimate with Failure Discounting, or MLEFD, estimates system reliability as the ratio of the number of successful trials to the total number of trials with this ratio being computed after application of failure discounting.

The other model was developed by H. Chernoff and W.M. Woods and is a derivation of an exponential single phase reliability estimate. This model employs linear regression techniques to estimate reliability after any change has been made to the unit of hardware under test. Because of this, the exponential regression model is able to track changing system reliability without the benefit of failure discounting although the use of failure discounting can enhance the model's performance.

The means of evaluating these two models was a Monte Carlo simulation. The simulation used was originally written by Captain James Drake and was modified so that fixed phase reliabilities could be modeled. The fixed phase reliability modification allows the user of the simulation to evaluate the performance of the two reliability growth models on any pattern of actual reliability growth desired. Thus, periods of declining reliability, periods of no reliability growth, and convexly growing reliability among others can be easily modeled and the performance of the two growth models analyzed.

Eight varying reliability growth patterns were analyzed. Both of the reliability growth models were evaluated against these eight patterns and under 13 different failure discounting combinations including no failure discounting. Each model, pattern and failure discounting combination was replicated 500 times. The ability of each model to accurately track the actual system reliability was evaluated along with an examination of the stability of the estimate produced by the model.

B. CONCLUSIONS

The evaluation of the two reliability growth models on the eight different reliability growth patterns leads to several general conclusions. The most obvious conclusion is that the use of either one of these growth models is superior to the standard single phase maximum likelihood estimate. Both growth models were far more accurate in terms of estimating the true system reliability and the variance associated with the model estimates was less than that produced by the single phase MLE.

The MLEFD is particularly sensitive to the choice of failure discounting procedure and parameters. This sensitivity is made use of in order to allow the model to track changing reliability. However, this property also places a premium on the ability of the test engineer to correctly select the proper discounting method and parameters.

The MLEFD did very well when the actual system reliability was constant. Constant system reliability is seldom present in actual reliability testing, however. The MLEFD also did well when the true reliability was increasing at a constant rate. Although the exponential regression model more closely followed the actual reliability growth pattern, the MLEFD would be preferred in this instance because of its smaller variability. The MLEFD did best with these patterns when the standard discount method was used with the discount fraction equal to 0.25 and the discount interval equal to 6 or 15.

The exponential regression model, although more variable in the early stages of testing, demonstrated the ability to accurately track each of the eight reliability growth patterns evaluated, even without the use of failure discounting. This model is also much less sensitive to the choice of discount method although its performance can be somewhat enhanced through failure discounting. The MLEFD, on the other hand, failed to track the decline in reliability in Pattern II.

Therefore, the exponential regression model is recommended for situations where a proven means of failure discounting does not exist. If one is confident of the failure discounting methodology then the MLEFD may be more advantageous due to the stability of the estimates produced by it. Regardless of the degree of confidence one has in the failure discounting procedures, the exponential regression model is recommended if abnormal reliability growth is anticipated. Abnormal reliability growth can be defined as a reliability pattern that has periods of decreasing or constant reliability in the middle of a generally increasing growth rate. The exponential regression model demonstrated superiority to the MLEFD in its ability to track differing patterns of growth.

The Lloyd discount method, in its original form, caused both models to overestimate the actual system reliability. Therefore, use of this method with these two reliability growth models is not recommended. A modified version of the Lloyd method which employs a discount interval demonstrated superior performance in some patterns to the original Lloyd discounting method.

C. RECOMMENDATIONS FOR FURTHER STUDY

The following are recommendations for further study and possible improvement to the models introduced in the study:

- This study addressed only discrete reliability growth models. A similar study analyzing the characteristics of continuous reliability growth models should be conducted. This would necessitate the construction of a different simulation.
- Currently, the failure discounting mechanism used in the simulation employed in this analysis only allows the user to set parameter values at the beginning. This should be modified so that discount parameters can be changed during the course of the simulation.
- The failure discounting mechanism should be modified so that different parameters can be applied to different failure causes. It may be desired that system failures caused by failure cause A, for example, be discounted less than those caused by failure cause B. This is not possible with the current simulation.
- Further analysis is required of the modified Lloyd failure discounting method. Many more simulation runs with varying reliability growth patterns and different

combinations of discount parameters are required before any solid conclusions can be reached.

- A weighted least squares method that weights recent data more heavily than previous data should be analyzed to determine its ability to track reliability growth patterns with sudden changes in slope or direction.

APPENDIX A. DERIVATION OF THE EXPONENTIAL REGRESSION MODEL

This model was developed by H. Chernoff and W.M. Woods. The derivation has been presented in previous works, most recently by W.M. Woods in a paper entitled "Reliability Growth Models" and by Captain James Drake in a thesis entitled "Discrete Reliability Growth Models using Failure Discounting". The exponential regression model is based on the exponential single phase reliability estimate. The exponential single phase estimate may be expressed mathematically as follows:

$$R = 1 - e^{-A}$$

The exponential regression model uses linear regression to estimate A so

$$A = \alpha + \beta k$$

where k is the testing phase being used to compute system reliability. Thus, the exponential regression model has the following formula:

$$\tilde{R} = 1 - e^{-(\alpha+\beta k)} \quad , \; k = 1,2,3,...$$

The estimates, $\hat{\alpha}_k$ and $\hat{\beta}_k$, for α and β at the end of the kth phase are obtained using the techniques of linear regression and an unbiased estimate for $(\alpha + \beta k)$. Let F denote the total number of failures possible during the kth phase and let j equal the failure number in phase k such that $j = 1,2,3,...,F$. N_{jk} equals the number of trials between the (j-1)st failure and the jth failure, including the jth failure, in the kth phase.

An unbiased estimate of $(\alpha + \beta k)$ using the jth set of trials in phase k is given by:

$$Y_{jk} = 1 + \frac{1}{2} + ... + \frac{1}{N_{jk} - 1} \quad if\ N_{jk} \geq 2$$

If $N_{jk} = 1$ (the first test was a failure) then Y_{jk} is set equal to zero. The least squares estimates, $\hat{\alpha}_k$ and $\hat{\beta}_k$, for α and β at the kth phase are then:

$$\hat{\beta}_k = [\sum_{i=0}^{k}(i - \overline{K})\overline{Y}_i] \div [\sum_{i=0}^{k}(i - \overline{K})^2]$$

$$\hat{\alpha}_k = \overline{\overline{Y}} - \hat{\beta}_k \overline{K}$$

where,

$$\overline{Y}_i = (Y_{1i} + Y_{2i} + \ldots + Y_{Fi})/F$$

$$\overline{K} = (1 + 2 + \ldots + k)/k$$

$$\overline{\overline{Y}} = (\overline{Y}_1 + \overline{Y}_2 + \ldots + \overline{Y}_k)/k$$

Substituting these estimates for α_k and β_k into the reliability equation yields the following estimate of reliability for phase k:

$$\tilde{R}_k = 1 - e^{-(\hat{\alpha}_k + \hat{\beta}_k k)} \quad for \;\; k > 1$$

APPENDIX B. FORTRAN CODE FOR SIMULATION

```
*************************************************************************
*                                                                       *
*           DISCRETE RELIABILITY GROWTH SIMULATION                      *
*                                                                       *
*               PROGRAMMED BY JAMES E DRAKE                             *
*                                                                       *
*               MODIFIED BY JAMES D CHANDLER                            *
*                                                                       *
*                LAST MODIFIED 29 FEB 1988                              *
*                                                                       *
*                                                                       *
*  THE FOLLOWING EXTERNAL FILES ARE USED BY THE PROGRAM                 *
*  INPUT  : DATA AND PARAMETER INPUT FILE (DEVICE # 10)                 *
*  THESIS : OUTPUT FILE CONTAINING INTERMEDIATE COMPUTATIONS            *
*           (DEVICE # 20)                                               *
*  RELIAB: OUTPUT FILE CONTAINING FINAL RESULTS OF THE SIMULATION       *
*           (DEVICE # 30)                                               *
*  EST    : OUTPUT FILE CONTAINING EACH PHASE ESTIMATE FOR EACH         *
*           REPLICATION OF THE WOODS WEIGHTED AVERAGE ESTIMATE          *
*           (DEVICE # 40)                                               *
*  MLEWD  : OUTPUT FILE CONTAINING MLE ESTIMATES USING DISCOUNTING      *
*           FOR EACH PHASE AND EACH REPLICATION                         *
*           (DEVICE # 50)                                               *
*  MLESP  : OUTPUT FILE CONTAINING MLE ESTIMATE FOR EACH SINGLE PHASE   *
*           AND ALL REPLICATIONS USING NO DISCOUNTING                   *
*           (DEVICE # 60)                                               *
*  REGEST : OUTPUT FILE CONTAINING EACH PHASE ESTIMATE FOR EACH         *
*           REPLICATION OF THE EXPONENTIAL REGRESSION ESTIMATE          *
*           (DEVICE # 70)                                               *
*                                                                       *
*  THE FOLLOWING IS A LIST OF KEY ARRAYS USED IN THE SIMULATION         *
*                                                                       *
*  A      : MAIN WORKING ARRAY CONTAINS PROBABILITY OF SUCCESS FOR      *
*           EACH FAILURE CAUSE, NUMBER OF TRIALS UNTIL FAILURE FOR      *
*           EACH FAILURE CAUSE AND THE SYSTEM, CAUSE OF FAILURE,        *
*           PHASE NUMBER, ADJUSTED NUMBER OF TRIALS AND ADJUSTED        *
*           NUMBER OF FAILURES                                          *
*           DIMENSION ( ((2*#CAUSES)+7),#FAILURES )                     *
*  NFAPH  : CONTAINS THE NUMBER OF FAILURES IN EACH PHASE               *
*           DIMENSION (1,#PHASES)                                       *
*  NFCAUS : BINARY ARRAY USED TO DETERMINE IF A FAILURE OCCURRED IN     *
*           A PHASE                                                     *
*           DIMENSION ( 1,#FAILURE CAUSES)                              *
*  NTRIAL : CONTAINS THE NUMBER OF TRIALS SINCE LAST FAILURE OR         *
*           DISCOUNTING FOR EACH FAILURE CAUSE                          *
*           DIMENSION ( 1,#FAILURE CAUSES )                             *
*  PHREST : RECORDS THE PHASE ESTIMATE FOR EACH ESTIMATOR WITHIN A      *
*           SINGLE REPLICATION                                          *
*           DIMENSION (4,#PHASES)                                       *
*           ROW 1 : WOODS WEIGHTED AVERAGE ESTIMATE                     *
*           ROW 2 : MLE WITH DISCOUNTING                                *
```

```
*           ROW 3 : SINGLE PHASE MLE                                     *
*           ROW 4 : EXPONENTIAL REGRESSION ESTIMATE                      *
*  AREL   : CONTAINS ACTUAL SYSTEM RELIABILITY IN EACH PHASE             *
*           DIMENSION (1,#PHASES)                                        *
*  YJK    : CONTAINS YJK VALUES UP TO 1000                               *
*           DIMENSION (1,1000)                                           *
*  CUMSF  : CONTAINS THE NUMBER OF SUCCESS AND FAILURES FOR EACH         *
*           FAILURE CAUSE (USED WITH WOODS WEIGHTED AVERAGE EST.)        *
*           DIMENSION (3,#FAILURE CAUSES)                                *
*           ROW 1 : NUMBER OF FAILURES                                   *
*           ROW 2 : NUMBER OF SUCCESSES                                  *
*           ROW 3 : ADJUSTED NUMBER OF SUCCESSES                         *
*  REG    : ARRAY USED TO COMPUTE THE EXPONENTIAL REGRESSION ESTIMATE    *
*           DIMENSION (5,#PHASES)                                        *
*           ROW 1 : K BAR                                                *
*           ROW 2 : Y BAR                                                *
*           ROW 3 : Y BAR FOR THE PHASE                                  *
*           ROW 4 : B HAT                                                *
*           ROW 5 : A HAT                                                *
*                                                                        *
*  THE REMAINING ARRAYS ARE USED TO COMPUTE THE MEAN AND VARIANCE        *
*  OF EACH ESTIMATE AT EACH PHASE. THEY ALL HAVE THE SAME DIMENSIONS     *
*  AND STRUCTURE                                                         *
*           DIMENSION (4,#PHASES)                                        *
*           ROW 1 : RUNNING SUM OF ESTIMATES                             *
*           ROW 2 : RUNNING SUM OF SQUARED ESTIMATES                     *
*           ROW 3 : MEAN OF THE ESTIMATES                                *
*           ROW 4 : STANDARD DEVIATION OF THE ESTIMATES                  *
*                                                                        *
*  EST    : VALUES FOR THE WOODS WEIGHTED AVERAGE ESTIMATE               *
*  MLEWD  : VALUES FOR THE MLE WITH DISCOUNTING                          *
*  MLESP  : VALUES FOR THE SINGLE PHASE MLE                              *
*  REGEST : VALUES FOR THE EXPONENTIAL REGRESSION ESTIMATE
*                                                                        *
**************************************************************************


C  DEFINE AND DIMENSION VARIABLES

      PARAMETER (NR=50,NC=200)
      INTEGER REP,CUMSF,DISOPT,FRELOP,LDI,ALD
      REAL*4 MIN
      REAL*8 DSEED,MLESP,MLEWD,EST,EUL
      DIMENSION NFAPH(NR),A(NR,NC),NFCAUS(NR),NTRIAL(NR),PHREST(4,NR),ES
     CT(4,NR),MLEWD(4,NR),MLESP(4,NR),REGEST(4,NR),AREL(NR),YJK(1000),CU
     CMSF(3,NR),REG(5,NR)

C  READ IN THE NUMBER OF CAUSES TO BE USED ( NCAUSE ) AND THE NUMBER
C  OF PHASES ( NPHASE ) IN THE TEST

      READ(10,*) NCAUSE
      READ(10,*) NPHASE

C  CHECK IF FIXED RELIABILITY OPTION IS DESIRED. FIX EULER'S NUMBER.
```

```
      READ(10,*) FRELOP
      EUL = 0.5772156648

C  CREATE VARIABLES FOR THE ROW INDICES OF THE WORKING MATRIX ( A )
C    IPHASE: PHASE
C    ISYSPR: ACTUAL COMPONENT RELIABILITY
C    INTR: NUMBER OF TRIALS UP TO AND INCLUDING FAILURE
C    IFAILC: CAUSE OF THE FAILURE
C    IADJF: ADJUSTED NUMBER OF FAILURES                                ED
C           AFTER DISCOUNTING HAS BEEN APPLIED
C    IADJT: ADJUSTED NUMBER OF TRIALS AFTER DISCOUNTING HAS BEEN APPLIED
C    IYJK: YJK COMPUTED ON THE ADJUSTED NUMBER OF TRIALS

      IPHASE = (2*NCAUSE)+1
      ISYSPR = IPHASE +1
      INTR = ISYSPR + 1
      IFAILC = INTR + 1
      IADJF = IFAILC + 1
      IADJT = IADJF + 1
      IYJK = IADJT + 1


C  READ IN THE NUMBER OF FAILURES IN EACH PHASE ( NFAPH(I) ) AND
C  COMPUTE THE TOTAL NUMBER OF FAILURES IN THE TEST ( NFAIL )

      NFAIL = 0
      DO 10 I=1,NPHASE
           READ(10,*) NFAPH(I)
           NFAIL = NFAIL + NFAPH(I)
 10   CONTINUE

C  INPUT THE PROBABILITY OF SUCCESS IN A SINGLE TRIAL FOR EACH CAUSE
C  IN EACH PHASE IF FRELOP EQUALS ONE.

      IF (FRELOP .EQ. 1) THEN
        DO 15 I=1,NCAUSE
           DO 15 J=1,NFAIL
             READ(10,*) A(I,J)
 15     CONTINUE
      ELSE

C  INPUT THE PROBABILITY OF SUCCESS IN A SINGLE TRIAL FOR EACH CAUSE
C  IN THE FIRST PHASE IF FRELOP EQUALS ZERO.

      DO 20 I=1,NCAUSE
           READ(10,*) A(I,1)
 20   CONTINUE

      ENDIF

C  INPUT THE REMAINING VARIABLES , THE NUMBER OF SUCCESSFUL TRIALS
C  BEFORE A DISCOUNT IS APPLIED (N); THE DISCOUNT FACTOR (R); THE SEED
C  FOR THE RANDOM NUMBER GENERATOR, GGUBFS, (DSEED); RELIABILITY
C  GROWTH FRACTION (FRIMP); TRIGGER FOR PRINTING INTERMEDIATE OUTPUT
C  (IOPT)
```

```
C  TRIGGERS FOR SAVING EACH ESTIMATE AT EACH PHASE FOR EACH ESTIMATOR
C     IOPT1 : WOODS WEIGHTED AVERAGE MODEL
C     IOPT2 : MLE WITH DISCOUNTING
C     IOPT3 : SINGLE PHASE MLE
C     IOPT4 : EXPONENTIAL REGRESSION MODEL
C   DISCOUNTING OPTION TRIGGER (DISOPT); LLOYD FAILURE DISCOUNTING
C   PARAMETER (GAMMA); LLOYD DISCOUNT INTERVAL

      READ(10,*) N
      READ(10,*) R
      READ(10,*) DSEED
      READ(10,*) FRIMP
      READ(10,*) NREP
      READ(10,*) IOPT
      READ(10,*) IOPT1
      READ(10,*) IOPT2
      READ(10,*) IOPT3
      READ(10,*) IOPT4
      READ(10,*) DISOPT
      READ(10,*) GAMA
      READ(10,*) LDI

      XNREP = NREP
      DSEED1 = DSEED

C  INITIALIZE THE ARRAYS USED TO COMPUTE THE MEAN AND STANDARD DEVIATION
C  OF EACH ESTIMATOR

      DO 30 J=1,NPHASE
           DO 30 I=1,4
                EST(I,J) = 0.0
                MLEWD(I,J) = 0.0
                MLESP(I,J) = 0.0
                REGEST(I,J) = 0.0
                PHREST(I,J) = 0.0
 30   CONTINUE

C  COMPUTE AND STORE THE YJK VALUES UP TO 1000

      YJK(1) = 0.0
      DO 40 I=1,999
           YJK(I+1) = YJK(I) + 1.0/I
 40   CONTINUE

C  COMPUTE AND STORE K BAR FOR THE EXPONENTIAL REGRESSION MODEL

      SUM = 0.0
      DO 50 I=1,NPHASE
         SUM = SUM + I
         REG(1,I) = SUM/I
 50   CONTINUE

C  MAJOR REPETITION OF THE SIMULATION LOOP

      DO 500 REP=1,NREP
```

```
C  INITIALIZE FAILURE CAUSE VECTOR (NFCAUS) AND (CUMSF)
C  COMPUTE THE INITIAL SYSTEM RELIABILITY

      REL = 1.
      DO 60 I=1,NCAUSE
           NFCAUS(I) = 0
           REL = REL * A(I,1)
           DO 60 J=1,3
              CUMSF(J,I) = 0
 60   CONTINUE

C  INITIALIZE COLUMN (FAILURE # ) COUNTER FOR THE WORKING ARRAY (A)

      J = 1

C  LOOP TO COMPUTE THE NUMBER OF TRIALS UP TO AND INCLUDING FAILURE
C  AND THE CAUSE OF FAILURE FOR EACH FAILURE IN EACH PHASE

      DO 130 K=1,NPHASE

C  SKIP ACTUAL COMPONENT RELIABILITY COMPUTATION AFTER FIRST REP
C  AND FOR FIRST FAILURE

        IF(J.EQ.1) GOTO 75
        IF(REP.GT.1) GOTO 75
        REL = 1.

C  IF FIXED RELIABILITY OPTION IS SELECTED THEN PHASE RELIABILITIES
C  ARE COMPUTED AS FOLLOWS

        IF (FRELOP .EQ. 1) THEN
          DO 65 I=1,NCAUSE
            REL = REL*A(I,J)
            NFCAUS(I) = 0
 65       CONTINUE
        ELSE

C  COMPUTE NEW ACTUAL RELIABILITY FOR THE COMPONENT IN PHASE K

        DO 70 I=1,NCAUSE

C  INCREASE CAUSE PR(SUCCESS) IF IT CAUSED FAILURE IN THE PREVIOUS PHASE
C  COMPUTE NEXT PHASE RELIABILITY AND REINITIALIZE NFCAUS (NOT USED IF
C  FIXED PHASE RELIABILITY OPTION IS SELECTED).

          IF(NFCAUS(I).EQ.1) THEN
          A(I,J) = A(I,(J-1)) + ((1. - A(I,(J-1)))*FRIMP)
          ELSEIF(NFCAUS(I).NE.1) THEN
          A(I,J) = A(I,(J-1))
          ELSE
          ENDIF
          REL = REL*A(I,J)
          NFCAUS(I) = 0
 70     CONTINUE
```

```
      ENDIF

 75   J1 = 1
      TRTOT = 0.0

C  COMPUTE THE NUMBER OF TRIALS UP TO AND INCLUDING FAILURE AND THE
C  CAUSE OF FAILURE FOR EACH FAILURE IN THE PHASE

      DO 120 L=1,NFAPH(K)
        IF(REP.GT.1) GOTO 90
        IF(J1.EQ.1) GOTO 85
        IF (FRELOP .EQ. 1) GOTO 85
        DO 80 I=1,NCAUSE
          A(I,J) = A(I,(J-1))
 80     CONTINUE
 85     A(ISYSPR,J) = REL
        A(IPHASE,J) = K
 90     MIN = 7.2E75
        DO 110 I=1,NCAUSE

C  ASSIGN # TRIALS FOR CAUSES WITH PR(SUCCESS) = 0 OR 1

          IF(A(I,J).GE.1.) THEN
            A((I+NCAUSE),J) = 7.2E75
            GOTO 100
          ELSEIF(A(I,J).EQ.0.) THEN
            A((I+NCAUSE),J) = 1.
            GOTO 100
          ELSE
          ENDIF

C  CONVERT UNIFORM (0,1) RANDOM VARIABLE TO GEOMETRIC (# TRIALS UNTIL
C  FAILURE ) FOR EACH FAILURE CAUSE. RECORD THE MIN # TRIALS FOR THE
C  CAUSES AS THE SYSTEM # TRIALS UP TO AND INCLUDING FAILURE AND
C  RECORD THE FAILURE CAUSE

          A((I+NCAUSE),J) = INT(1.+(LOG(GGUBFS(DSEED))/LOG(A(I,J))))
 100      IF(A((I+NCAUSE),J).LE.MIN) THEN
            MIN = A((I+NCAUSE),J)
            IMIN = I
          ELSE
          ENDIF
 110    CONTINUE
        A(IFAILC,J) = IMIN
        NFCAUS(IMIN) = 1

C  COMPUTE THE TOTAL # OF TRIALS FOR THE MLE SINGLE PHASE ESTIMATE AND
C  INCREMENT FAILURE # COUNTERS

        A(INTR,J) = MIN
        TRTOT = TRTOT + A(INTR,J)
        J = J + 1
        J1 = J1 + 1
 120  CONTINUE
```

```
C  COMPUTE THE MLE ESTIMATE OF COMPONENT RELIABILITY FOR THIS PHASE AND
C  COMPUTE THE RUNNING SUM OF ESTIMATES AND THE SUM OF ESTIMATES SQUARED
C  FOR COMPUTATION OF THE MEAN AND STANDARD DEVIATION OF THE ESTIMATE

      PHREST(3,K) = (TRTOT - NFAPH(K))/TRTOT
      MLESP(1,K) = MLESP(1,K) + PHREST(3,K)
      MLESP(2,K) = MLESP(2,K) + (PHREST(3,K)**2)

 130  CONTINUE

C  INITIALIZE THE ADJUSTED NUMBER OF FAILURES TO 1 AND THE COUNT OF THE
C  NUMBER OF TRIALS SINCE FAILURE OR DISCOUNTING (NTRIALS(I) ) TO 0
C  IN PREPARATION FOR THE DISCOUNTING ROUTINE

      DO 140 J=1,NFAIL
           A(IADJF,J) = 1.
 140  CONTINUE

      DO 150 I=1,NCAUSE
           NTRIAL(I) = 0
 150  CONTINUE

C  DISCOUNTING ROUTINE REVIEWS ALL PAST FAILURES AND CAUSES TO DATE
C  AND DETERMINES IF THE DISCOUNTING CONDITIONS HAVE BEEN MET. COMPUTES
C  THE ADJUSTED FAILURES, THE ADJUSTED # OF TRIALS AND YJK

      J = 0
      DO 300 K=1,NPHASE
        DO 200 L=1,NFAPH(K)
          J = J + 1

C  UPDATES THE NUMBER OF TRIALS SINCE FAILURE OR DISCOUNTING FOR EACH
C  FAILURE CAUSE

          ICAUSE = INT(A(IFAILC,J)+.5)
          DO 160 I=1,NCAUSE
            IF(ICAUSE.EQ.I) THEN
              NTRIAL(I) = 0
            ELSEIF(ICAUSE.NE.I) THEN
              NTRIAL(I) = NTRIAL(I) + INT(A(INTR,J)+.5)
            ELSE
            ENDIF
 160      CONTINUE

 200  CONTINUE

C  CHOOSE DISCOUNTING METHOD TO BE USED

         IF(DISOPT.NE.2) GOTO 180

C  PERFORM LLOYD'S FAILURE DISCOUNTING METHOD

         DO 170 I=1,J
              I1 = INT(A(IFAILC,I)+.5)
              IF(NTRIAL(I1).EQ.0) THEN
                   A(IADJF,I) = 1.0
```

```
                    GOTO 170
                ELSE
                ENDIF

C  THIS IS THE MODIFIED LLOYD METHOD USING A DISCOUNT INTERVAL.  THE
C  ORIGINAL DISCOUNT METHOD MAY BE EMPLOYED BY SETTING LDI TO ONE.

            ALD = INT(NTRIAL(I1)/LDI)
            IF(ALD .EQ. 0) THEN
               A(IADJF,I) = 1.0
               GO TO 170
            ELSE
               A(IADJF,I) = 1.0 - ((1.-GAMA)**(1.0/ALD))
            ENDIF

 170        CONTINUE
            GOTO 210

C  PERFORMS STRAIGHT PERCENT FAILURE DISCOUNTING AND
C  COMPUTES THE ADJUSTED # OF FAILURES

 180         DO 190 I=1,J
               I1 = INT(A(IFAILC,I)+.5)
               IF(NTRIAL(I1).EQ.0) THEN
                 A(IADJF,I) = 1.
               ELSEIF(NTRIAL(I1).GE.N) THEN
                 A(IADJF,I) = A(IADJF,I)*((1.-R)**(NTRIAL(I1)/N))
               ELSE
               ENDIF
 190         CONTINUE

C  ADJUSTS THE # TRIALS SINCE FAILURE OR DISCOUNTING FOR THOSE CAUSES
C  THAT HAVE MET OR SURPASSED THE DISCOUNTING THRESHOLD
C  FOR THE STRAIGHT PERCENT DISCOUNTING METHOD

             DO 205 I=1,NCAUSE
               IF(NTRIAL(I).GE.N) NTRIAL(I) = MOD(NTRIAL(I),N)
 205         CONTINUE
 210        TADJT = 0.0
            TYJK = 0.0
            TPYJK = 0.0
            K1 = 0

            DO 215 I2=1,3
            DO 215 I=1,NCAUSE
               CUMSF(I2,I) = 0
 215           CONTINUE

C  COMPUTES THE ADJUSTED # OF TRIALS FROM THE ADJUSTED # OF FAILURES
C  AND COMPUTES THE SUM OF THE ADJUSTED # OF TRIALS FOR ESTIMATE COMP.

            PREL = 0.0
            LTRIAL = 0

C  IF ADJUSTED FAILURES ARE APPROACHING 0 THEN ADJUSTED TRIALS MUST
C  BE PRE-SET.
```

```
      DO 240 I=1,J

        IF(A(IADJF,I) .LE. .0000001) THEN
          A(IADJF,I) = .0000001
         ENDIF

        A(IADJT,I) = A(INTR,I)/A(IADJF,I)
        TADJT = TADJT + A(IADJT,I)

C  COMPUTE YJK FROM THE ADJUSTED # OF TRIALS AND STORE THE SUM FOR
C  ESTIMATE COMPUTATION, USE ARRAY FOR # TRIALS < 1000 AND APPROX. FOR
C  VALUES > 1000

        N1 = NINT(A(IADJT,I))
        IF(N1.LE.1000) THEN
            A(IYJK,I) = YJK(N1)
        ELSEIF(N1.GT.1000) THEN
            X=N1
            Q=12*X
            T=X+1
            S=X+2

        A(IYJK,I)=(EUL+(LOG(X))+(1/(2*X))-(1/(Q*T))-(1/(Q*T*S)))

        ELSE
        ENDIF

C  DETERMINE IF A PHASE BOUNDARY HAS BEEN REACHED TO BEGIN ESTIMATE
C  COMPUTATION

      IF(I.EQ.1) GOTO 225
      IF(A(IPHASE,I).NE.A(IPHASE,(I-1))) THEN

C  COMPUTE THE WOODS WEIGHTED AVERAGE ESTIMATE

      MAX = 0
      K1 = K1 + 1

C  DETERMINE THE FAILURE CAUSE WITH THE LARGEST # OF FAILURES

      DO 220 I1=1,NCAUSE
         IF(CUMSF(1,I1).GT.MAX) THEN
         MAX = CUMSF(1,I1)
         ICOL = I1
         ELSE
         ENDIF
 220  CONTINUE

C COMPUTE YJK VALUE FOR THE CURRENT PHASE ESTIMATE

      IF(CUMSF(1,ICOL).LE.1000) THEN
         AHATL = YJK(CUMSF(1,ICOL))
      ELSEIF(CUMSF(1,ICOL).GT.1000) THEN
         X = CUMSF(1,ICOL)
         Q=12*X
```

```
         T=X+1
         S=X+2

         AHATL=(EUL+(LOG(X))+(1/(2*X))-(1/(Q*T))-(1/(Q*T*S)))

      ELSE
      ENDIF
      IX = CUMSF(1,ICOL) + CUMSF(3,ICOL)
      IF(IX.LE.1000) THEN
         AHATU = YJK(IX)
      ELSEIF(IX.GT.1000) THEN
         X = IX
         Q=12*X
         T=X+1
         S=X+2

         AHATU=(EUL+(LOG(X))+(1/(2*X))-(1/(Q*T))-(1/(Q*T*S)))

      ELSE
      ENDIF

C COMPUTE CURRENT PHASE RELIABILITY ESTIMATE

      AHAT = AHATU - AHATL
      CREL = 1.0 - EXP(-AHAT)
      X = CUMSF(1,ICOL) + CUMSF(3,ICOL)

C  COMPUTE AND STORE THE WOODS WEIGHTED AVERAGE ESTIMATE

      PREL = ((LTRIAL*PREL)/X) + (((X-LTRIAL)*CREL)/X)
      LTRIAL = CUMSF(1,ICOL) + CUMSF(3,ICOL)

C  COMPUTE THE PHASE AND GLOBAL AVERAGE FOR YJK USED IN THE EXPONENTIAL
C  REGRESSION ESTIMATES ARE

      REG(2,K1) = TYJK/(I-1)
      REG(3,K1) = TPYJK/NFAPH(K1)
      TPYJK = 0.0

      ENDIF

C  COMPUTE THE NUMBER OF FAILURES AND SUCCESSES FOR EACH FAILURE CAUSE
C  USED IN THE WOODS WEIGHTED AVERAGE ESTIMATE

 225  ICAUSE = INT(A(IFAILC,I)+.5)
      DO 230 I1=1,NCAUSE
         CUMSF(2,I1) = CUMSF(2,I1) + INT(A(INTR,I) + .5)
         CUMSF(3,I1) = CUMSF(3,I1) + N1
 230  CONTINUE
      CUMSF(1,ICAUSE) = CUMSF(1,ICAUSE) + 1
      CUMSF(2,ICAUSE) = CUMSF(2,ICAUSE) - 1
      CUMSF(3,ICAUSE) = CUMSF(3,ICAUSE) - 1
          TPYJK = TPYJK + A(IYJK,I)
          TYJK = TYJK + A(IYJK,I)
 240    CONTINUE
```

```
C  REPEAT COMPUTATIONS FOR THE WOODS WEIGHTED AVERAGE ESTIMATE FOR THE
C  FINAL PHASE

      MAX = 0
      K1 = K1 + 1
      DO 245 I1=1,NCAUSE
         IF(CUMSF(1,I1).GT.MAX) THEN
         MAX = CUMSF(1,I1)
         ICOL = I1
         ELSE
         ENDIF
 245  CONTINUE

      IF(CUMSF(1,ICOL).LE.1000) THEN
         AHATL = YJK(CUMSF(1,ICOL))
      ELSEIF(CUMSF(1,ICOL).GT.1000) THEN
         X = CUMSF(1,ICOL)
         Q=12*X
         T=X+1
         S=X+2

         AHATL=(EUL+(LOG(X))+(1/(2*X))-(1/(Q*T))-(1/(Q*T*S)))

      ELSE
      ENDIF
      IX = CUMSF(1,ICOL) + CUMSF(3,ICOL)
      IF(IX.LE.1000) THEN
         AHATU = YJK(IX)
      ELSEIF(IX.GT.1000) THEN
         X = IX
         Q=12*X
         T=X+1
         S=X+2

         AHATU=(EUL+(LOG(X))+(1/(2*X))-(1/(Q*T))-(1/(Q*T*S)))

      ELSE
      ENDIF

      AHAT = AHATU - AHATL
      CREL = 1.0 - EXP(-AHAT)
      X = CUMSF(1,ICOL) + CUMSF(3,ICOL)


      PREL = ((LTRIAL*PREL)/X) + (((X-LTRIAL)*CREL)/X)
      LTRIAL = CUMSF(1,ICOL) + CUMSF(3,ICOL)
      REG(2,K1) = TYJK/(J)
      REG(3,K1) = TPYJK/NFAPH(K1)

        PHREST(1,K) = PREL

C  COMPUTE THE MLE ESTIMATE OF PHASE RELIABILITY USING DISCOUNTING

        PHREST(2,K) = (TADJT - J)/TADJT

C  COMPUTE THE EXPONENTIAL REGRESSION ESTIMATE BEGINNING WITH B HAT
```

```
        SUM = 0.0
        SUMS = 0.0
        IF (K.EQ.1) GOTO 252
        DO 250 I = 1,K
             SUM = SUM + ((I-REG(1,K))*REG(3,I))
             SUMS = SUMS + ((I-REG(1,K))**2)
 250    CONTINUE

        REG(4,K) = SUM/SUMS

C  COMPUTE A HAT

        REG(5,K) = REG(2,K) - (REG(4,K)*REG(1,K))

C  COMPUTE AND STORE THE EXPONENTIAL REGRESSION ESTIMATE

        PHREST(4,K) = 1.0 - EXP(-(REG(5,K) + (REG(4,K)*K)))
        IF(PHREST(4,K).LT.0.0) PHREST(4,K)=0.0
        GOTO 255
 252    PHREST(4,K) = 1.0 - EXP(-REG(3,1))
        IF(PHREST(4,K).LT.0.0) PHREST(4,K)=0.0

C  STORE THE RUNNING SUM OF THE ESTIMATES FOR THE CURRENT PHASE AND THE
C  RUNNING SUM OF THE ESTIMATES SQUARED FOR COMPUTATION OF THE MEAN AND
C  STANDARD DEVIATION OF EACH ESTIMATE FOR EACH RELIABILITY GROWTH
C  MODEL

 255    EST(1,K) = EST(1,K) + PHREST(1,K)
        EST(2,K) = EST(2,K) + (PHREST(1,K)**2)
        MLEWD(1,K) = MLEWD(1,K) + PHREST(2,K)
        MLEWD(2,K) = MLEWD(2,K) + (PHREST(2,K)**2)
        REGEST(1,K) = REGEST(1,K) + PHREST(4,K)
        REGEST(2,K) = REGEST(2,K) + (PHREST(4,K)**2)

C  STORE THE ACTUAL PHASE RELIABILITY

        AREL(K) = A(ISYSPR,J)

C  PRINT INTERMEDIATE OUTPUT IF REQUESTED AND THE NUMBER OF REPETITIONS
C  IS NOT GREATER THAN 5

        IF(IOPT.NE.1) GOTO 300
        IF(REP.GT.5) GOTO 300

      WRITE(20,1000) REP,K
 1000 FORMAT(T16,'REPETITION NUMBER: ',I4,'   PHASE NUMBER: ',I4)
      WRITE(20,1010) A(ISYSPR,J)
 1010 FORMAT(22X,'ACTUAL COMPONENT RELIABILITY: ',F7.5)
      WRITE(20,1020) PHREST(1,K)
 1020 FORMAT(20X,'PREDICTED COMPONENT RELIABILITY: ',F7.5)
      WRITE(20,1022) PHREST(2,K)
 1022 FORMAT(20X,'MLE ESTIMATE USING DISCOUNTING: ',F7.5)
      WRITE(20,1025) PHREST(3,K)
 1025 FORMAT(18X,'MLE ESTIMATE OF PHASE RELIABILITY: ',F7.5)
      WRITE(20,1027) PHREST(4,K)
```

```
 1027 FORMAT(14X,'REGRESSION ESTIMATE OF PHASE RELIABILITY: ',F7.5)
      WRITE(20,1030)
 1030 FORMAT(' ','     ')
      DO 260 I=1,NCAUSE
        WRITE(20,1035)I,A(I,J),A((I+NCAUSE),J)
 1035   FORMAT(12X,'CAUSE: ',I3,'   PR(SUCCESS): ',F7.6,'   # TRIALS: ',
     CF10.0)
 260  CONTINUE
      WRITE(20,1036)
 1036 FORMAT(' ','     ')
      WRITE(20,1040)
 1040 FORMAT(4X,'FAIL #',3X,'FAIL CAUSE',3X,'# TRIALS',3X,'ADJ # FAIL',3
     CX,'ADJ # TRIALS',7X,'YJK')
      DO 270 I=1,J
      WRITE(20,1050)I,A(IFAILC,I),A(INTR,I),A(IADJF,I),A(IADJT,I),A(IYJK
     C,I)
 1050 FORMAT(4X,I3,8X,F3.0,7X,F8.0,4X,F8.6,4X,F12.0,3X,F11.4)
 270  CONTINUE
      WRITE(20,1060)
 1060 FORMAT(' ',///)

 300  CONTINUE
C  PRINT EACH OF THE 3 ESTIMATES TO THEIR APPROPRIATE OUTPUT FILE
C  IF REQUESTED

      IF(IOPT1.NE.1) GOTO 401
 400  WRITE(40,2000) (PHREST(1,I), I=1,NPHASE)
 401  IF(IOPT2.NE.1) GOTO 402
      WRITE(50,2000) (PHREST(2,I), I=1,NPHASE)
 402  IF(IOPT3.NE.1) GOTO 403
      WRITE(60,2000) (PHREST(3,I), I=1,NPHASE)
 403  IF(IOPT4.NE.1) GOTO 500
      WRITE(70,2000) (PHREST(4,I), I=1,NPHASE)
 2000 FORMAT(' ',30(F7.6:1X))

 500  CONTINUE

C  UPON COMPLETION OF ALL REPETITIONS, COMPUTE THE MEAN AND STANDARD
C  DEVIATION OF EACH ESTIMATE FOR EACH PHASE SKIPPING COMPUTATIONS IF
C  ONLY ONE REPETITION IS REQUIRED

      IF (NREP.LE.1) GOTO 601

      DO 600 I=1,NPHASE
      EST(3,I) = EST(1,I)/XNREP
      MLEWD(3,I) = MLEWD(1,I)/XNREP
      MLESP(3,I) = MLESP(1,I)/XNREP
      REGEST(3,I) = REGEST(1,I)/XNREP
      EST(4,I) = SQRT((EST(2,I)-(XNREP*(EST(3,I)**2)))/(XNREP-1))
      MLEWD(4,I) = SQRT((MLEWD(2,I)-(XNREP*(MLEWD(3,I)**2)))/(XNREP-1))
      MLESP(4,I) = SQRT((MLESP(2,I)-(XNREP*(MLESP(3,I)**2)))/(XNREP-1))
      REGEST(4,I)=SQRT((REGEST(2,I)-(XNREP*(REGEST(3,I)**2)))/(XNREP-1))
 600  CONTINUE

C  PRINT THE FINAL OUTPUT TABLE TO A FILE
```

```
601  WRITE(30,3000)
3000 FORMAT('0',T47,'DISCRETE RELIABILITY GROWTH SIMULATION')
     WRITE(30,3010)
3010 FORMAT('-',T54,'MODEL PARAMETER SUMMARY')
     WRITE(30,3020) NCAUSE
3020 FORMAT('0',T47,'NUMBER OF POSSIBLE FAILURE CAUSES ',I4)
     IF (FRELOP .EQ. 1) GOTO 4000
     WRITE(30,3030)
3030 FORMAT('0',T38,'CAUSE NUMBER',T64,'SINGLE TRIAL PR( SUCCESS ) FOR
    CPHASE 1')
     DO 3050 M=1,NCAUSE
     WRITE(30,3040) M,A(M,1)
3040 FORMAT(' ',T43,I2,T79,F8.6)
3050 CONTINUE
     WRITE(30,3060) FRIMP
3060 FORMAT('0',T37,'FRACTION CAUSE RELIABILITY IMPROVES AFTER FAILURE
    C',F8.6)
5000 WRITE(30,3080) NPHASE
3080 FORMAT('-',T48,'NUMBER OF PHASES IN THE SIMULATION ',I2)
     WRITE(30,3090)
3090 FORMAT('0',T42,'PHASE NUMBER',T59,'NUMBER OF FAILURES IN THE FIRST
    C PHASE')
     DO 3110 M=1,NPHASE
     WRITE(30,3100) M,NFAPH(M)
3100 FORMAT(' ',T43,I2,T73,I2)
3110 CONTINUE
     WRITE(30,3120) NFAIL
3120 FORMAT('0',T51,'TOTAL NUMBER OF FAILURES ',I4)
     IF(DISOPT.EQ.2) GO TO 3160
     WRITE(30,3130)
3130 FORMAT('-',T38,'DISCOUNTING PERFORMED USING THE CONSTANT FRACTION
    CMETHOD')
     WRITE(30,3140) R
3140 FORMAT('0',T44,'FRACTION EACH FAILURE IS DISCOUNTED ',F8.6)
     WRITE(30,3150) N
3150 FORMAT(' ',T33,'NUMBER OF TRIALS AFTER A FAILURE BEFORE A DISCOUNT
    C IS APPLIED ',I4)
     GO TO 3190
3160 WRITE(30,3170)
3170 FORMAT('-',T44,'DISCOUNTING PERFORMED USING THE LLOYD METHOD')
     WRITE(30,3180) GAMA
3180 FORMAT('0',T39,'PERCENT C.I. ( USED AS DISCOUNT FRACTION ) ',F8.6
    C)
     WRITE(30,3185) LDI
3185 FORMAT('0',T50,'LLOYD DISCOUNT INTERVAL: ',I3)
3190 WRITE(30,3200) DSEED1
3200 FORMAT('-',T46,'RANDOM NUMBER SEED USED ',F15.2)
     WRITE(30,3210) NREP
3210 FORMAT('0',T37,'NUMBER OF REPETITIONS OF THE SIMULATION PERFORMED
    C',I7)
     WRITE(30,3220)
3220 FORMAT('1',T61,'ESTIMATOR:')
     WRITE(30,3230)
3230 FORMAT('0',T48,'SINGLE PHASE MLE WITHOUT DISCOUNTING')
     WRITE(30,3240)
3240 FORMAT('-',T60,'MEAN',T83,'ESTIMATE',T109,'95 %')
```

```
      WRITE(30,3250)
 3250 FORMAT(' ',T12,'PHASE NUMBER',T29,'ACTUAL RELIABILITY',T52,'PREDIC
     CTED RELIABILITY',T78,'STANDARD DEVIATION',T101,'CONFIDENCE INTERVA
     CL')

C  COMPUTE C.I. FOR SINGLE PHASE MLE

      DO 3270 M=1,NPHASE
      CI = (1.96*MLESP(4,M))/SQRT(XNREP)
      CIU = MLESP(3,M) + CI
      CIL = MLESP(3,M) - CI
      WRITE(30,3260) M,AREL(M),MLESP(3,M),MLESP(4,M),CIL,CIU
 3260 FORMAT('0',T17,I2,T34,F8.6,T58,F8.6,T82,F9.6,T99,'( ',F8.6,' , ',F
     C8.6,' )')
 3270 CONTINUE
      WRITE(30,3220)
      WRITE(30,3280)
 3280 FORMAT('0',T42,'MAX LIKELIHOOD ESTIMATE USING DISCOUNTED FAILURES'
     C)
      WRITE(30,3240)
      WRITE(30,3250)

C  COMPUTE C.I. FOR MLE WITH DISCOUNTING

      DO 3290 M=1,NPHASE
      CI = (1.96*MLEWD(4,M))/SQRT(XNREP)
      CIU = MLEWD(3,M) + CI
      CIL = MLEWD(3,M) - CI
      WRITE(30,3260) M,AREL(M),MLEWD(3,M),MLEWD(4,M),CIL,CIU
 3290 CONTINUE
      WRITE(30,3220)
      WRITE(30,3300)
 3300 FORMAT('0',T38,'WEIGHTED AVERAGE ESTIMATE USING FAILURE DISCOUNTIN
     CG')
      WRITE(30,3240)
      WRITE(30,3250)

C  COMPUTE C.I. FOR WOODS WEIGHTED AVERAGE ESTIMATES

      DO 3310 M=1,NPHASE
      CI = (1.96*EST(4,M))/SQRT(XNREP)
      CIU = EST(3,M) + CI
      CIL = EST(3,M) - CI
      WRITE(30,3260) M,AREL(M),EST(3,M),EST(4,M),CIL,CIU
 3310 CONTINUE

      WRITE(30,3220)
      WRITE(30,3320)
 3320 FORMAT('0',T43,'REGRESSION ESTIMATE USING DISCOUNTED FAILURES')
      WRITE(30,3240)
      WRITE(30,3250)

C  COMPUTE C.I. FOR EXPONENTIAL REGRESSION ESTIMATES

      DO 3330 M=1,NPHASE
      CI = (1.96*REGEST(4,M))/SQRT(XNREP)
```

```
      CIU = REGEST(3,M) + CI
      CIL = REGEST(3,M) - CI
      WRITE(30,3260) M,AREL(M),REGEST(3,M),REGEST(4,M),CIL,CIU
3330  CONTINUE
      WRITE(30,3340)
3340  FORMAT('1',T59,'RECAPITULATION'//)
      WRITE(30,3350)
3350  FORMAT('-',T3,'PHASE',T11,'ACTUAL',T28,'MEAN',T38,'EST',T53,'MEAN'
     C,T63,'EST',T78,'MEAN',T88,'EST',T103,'MEAN',T113,'EST')
      WRITE(30,3360)
3360  FORMAT(' ',T11,'RELIAB',T28,'WGT',T38,'STD',T53,'MLE',T63,'STD',T7
     C7,'PHASE',T88,'STD',T103,'REG',T113,'STD')
      WRITE(30,3370)
3370  FORMAT(' ',T28,'AVG',T35,'DEVIATION',T53,'W/D',T60,'DEVIATION',T78
     C,'MLE',T85,'DEVIATION',T103,'EST',T110,'DEVIATION')
      WRITE(30,3375)
3375  FORMAT(' ',T28,'EST'/)
      DO 650 I=1,NPHASE
        WRITE(30,3380)I,AREL(I),EST(3,I),EST(4,I),MLEWD(3,I),MLEWD(4,I),
     CMLESP(3,I),MLESP(4,I),REGEST(3,I),REGEST(4,I)
3380  FORMAT('0',T4,I3,T11,F7.6,T26,F7.6,T36,F7.6,T51,F7.6,T61,F7.6,T76,
     CF7.6,T86,F7.6,T101,F7.6,T111,F7.6)
650   CONTINUE
      GO TO 6000

4000  WRITE(30,4010)
4010  FORMAT(1X,//,T50,'FIXED PHASE RELIABILITY OPTION')
      WRITE(30,4020)
4020  FORMAT('-',T38,'PHASE NUMBER',T78,'ACTUAL RELIABILITY')
      DO 4030 M=1,NPHASE
        WRITE(30,4040) M,AREL(M)
4040  FORMAT('0',T41,I2,T83,F8.6)
4030  CONTINUE
      GO TO 5000
6000  CONTINUE

      STOP
      END
```

APPENDIX C. GRAPHICAL RESULTS

This appendix contains the results of many of the various runs done with the simulation. These results appear in tabular format in the output file but have been reduced to graphical form for ease of understanding. Below each graph of the estimates produced by the growth models are graphs of the standard deviations of each estimate by phase. This appendix is ordered by reliability growth pattern with Pattern I first and the remainder following sequentially.

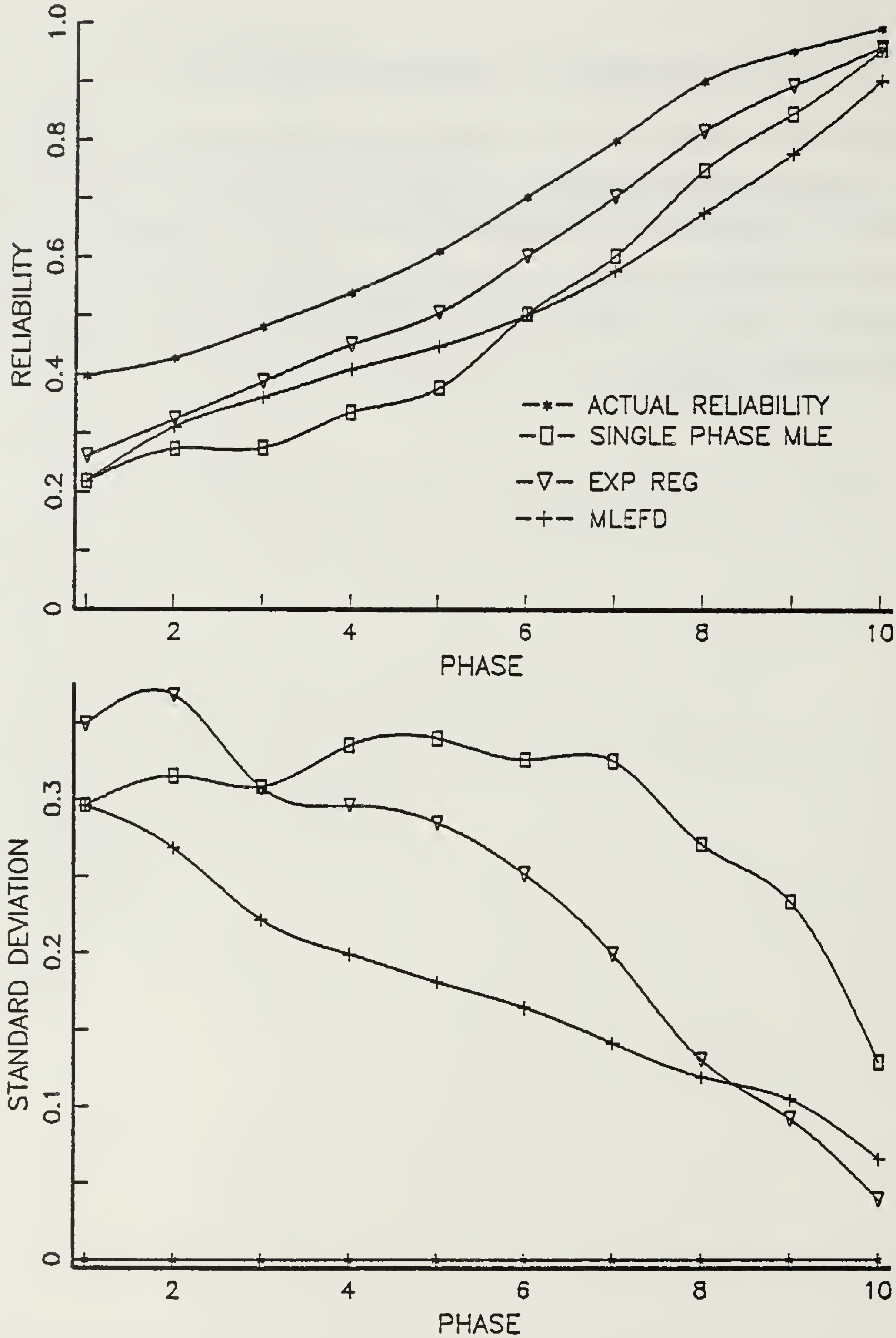

Figure 27. Pattern I, No Discounting

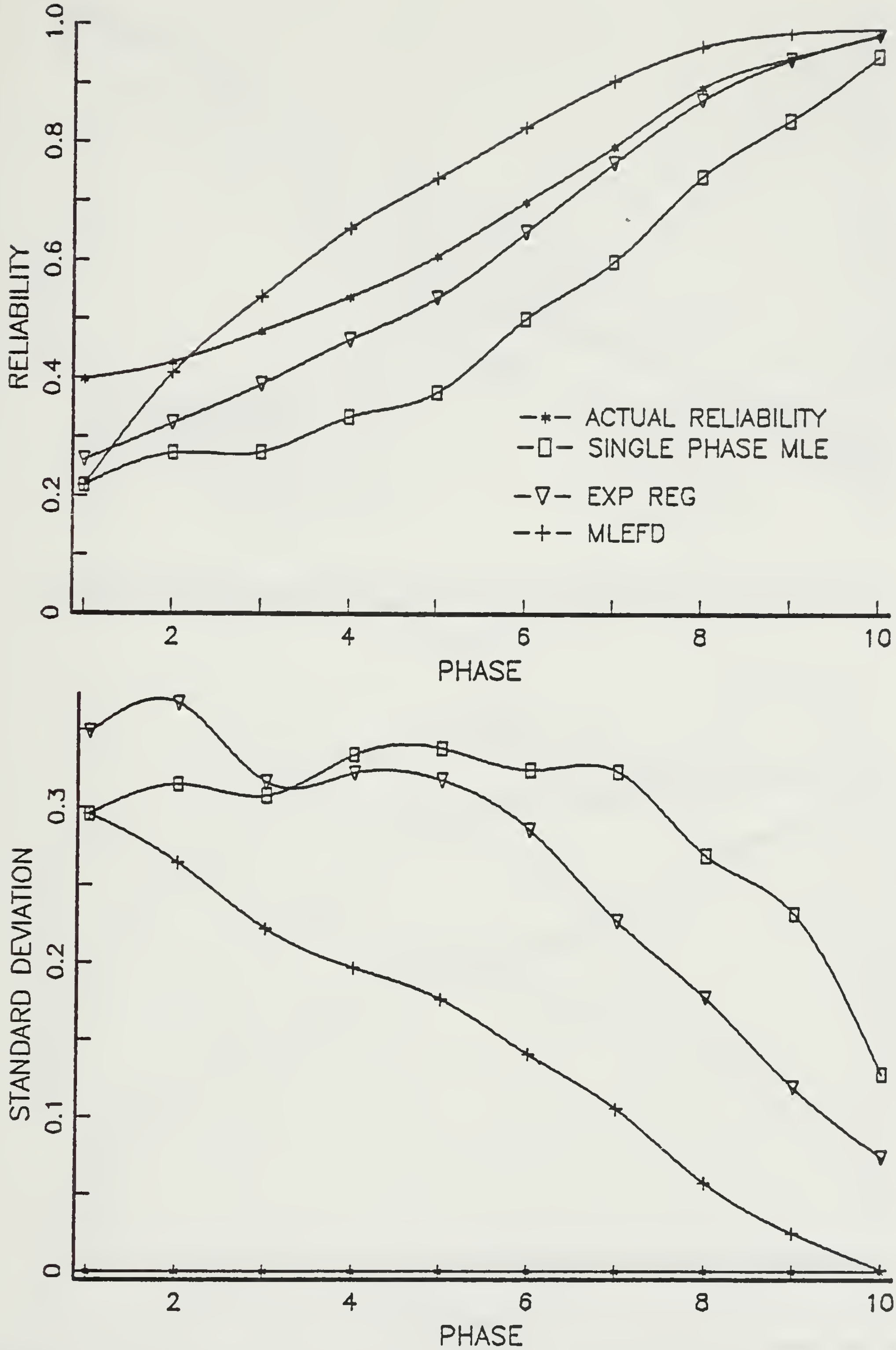

Figure 28. Pattern I, F = .25, I = 1

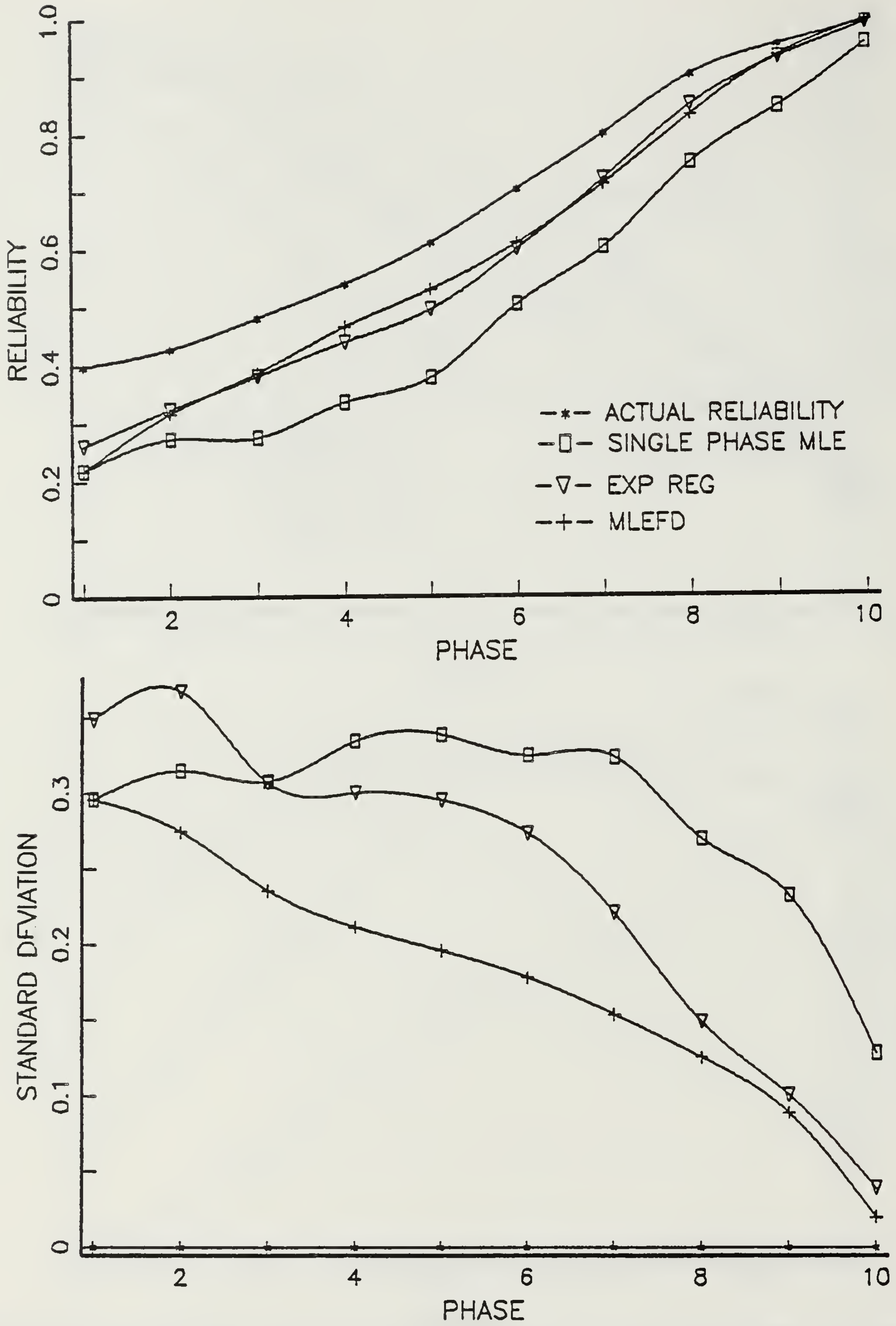

Figure 29. Pattern I, F = .25, I = 3

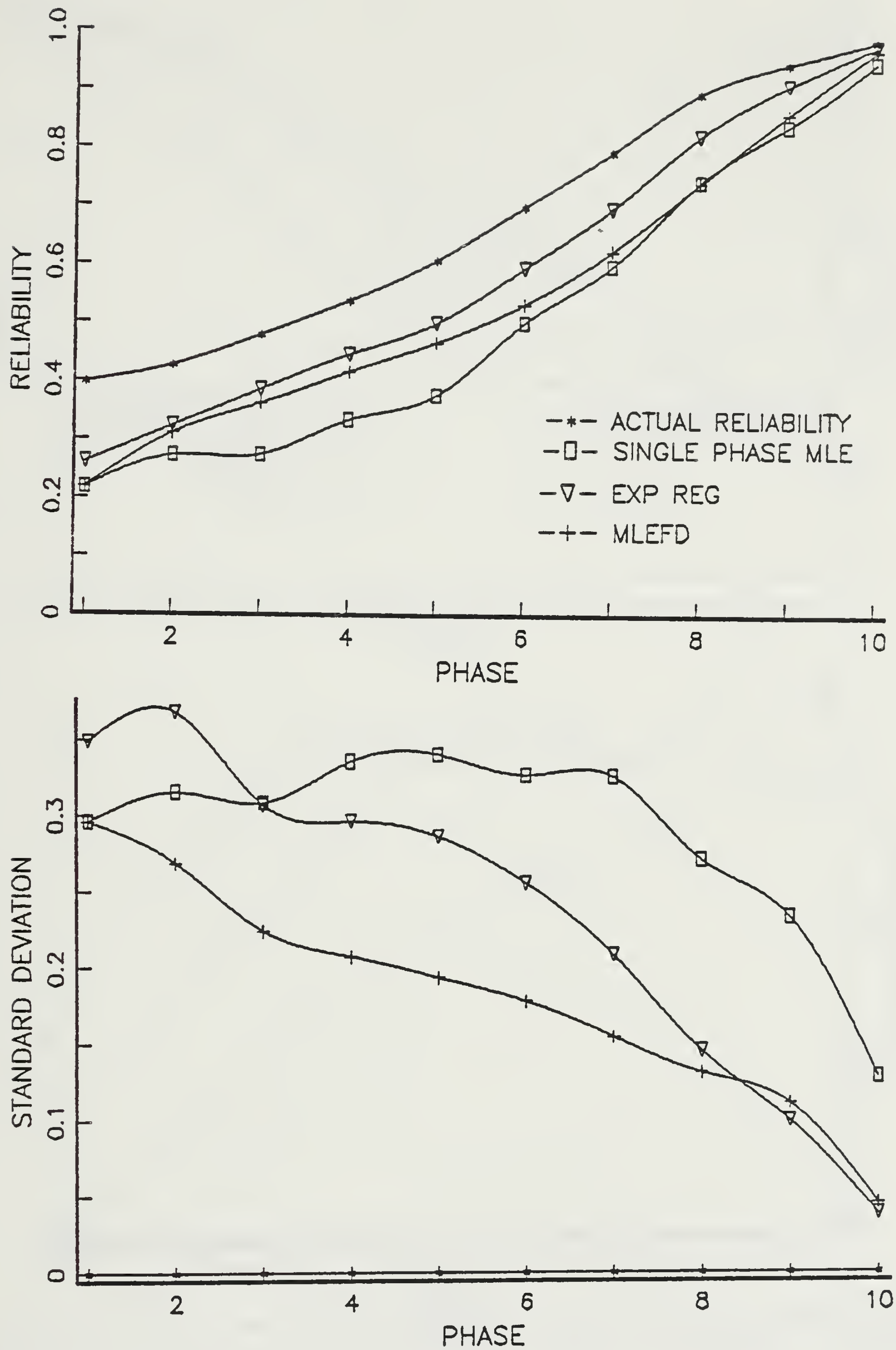

Figure 30. Pattern I, F = .25, I = 6

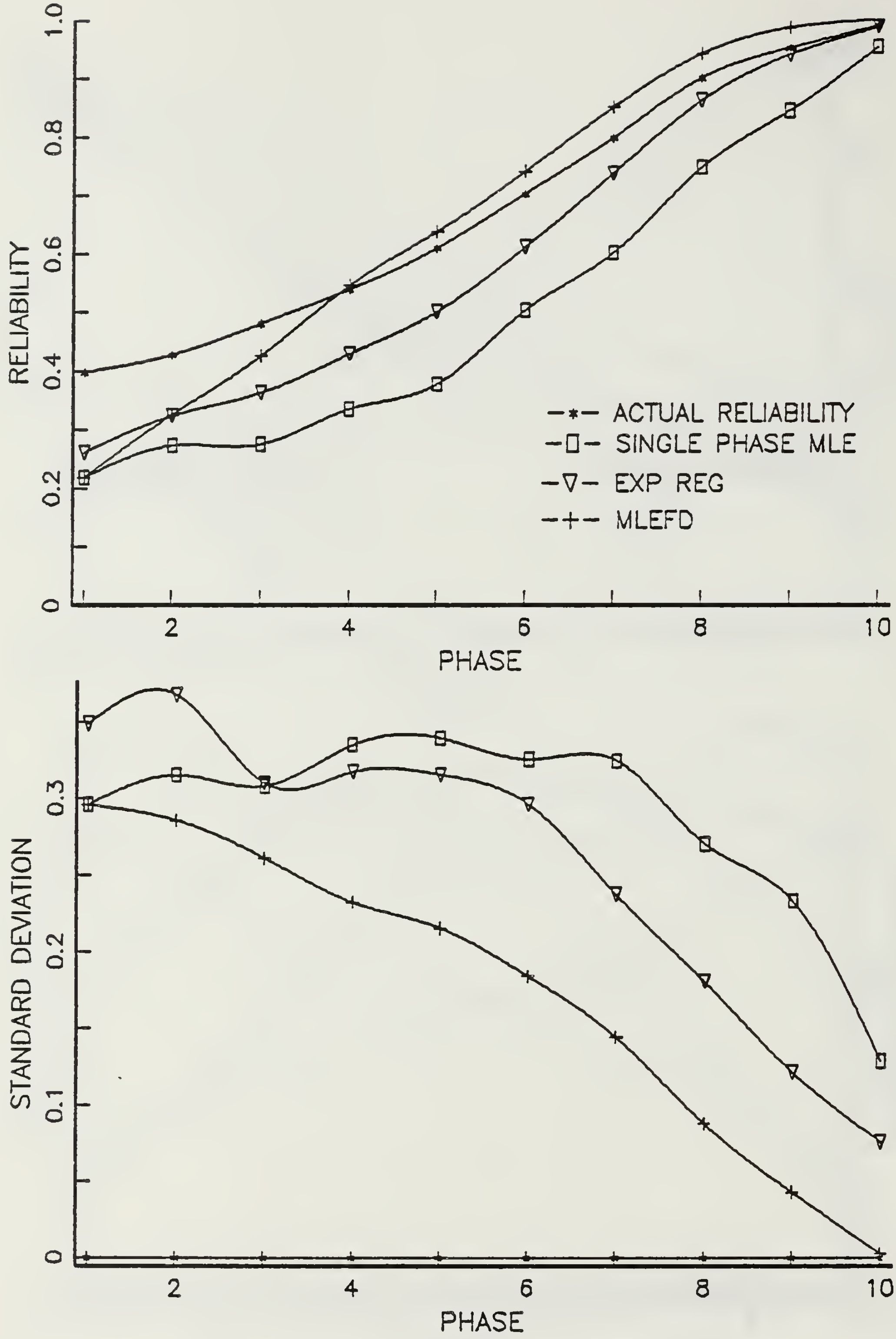

Figure 31. Pattern I, F = .50, I = 3

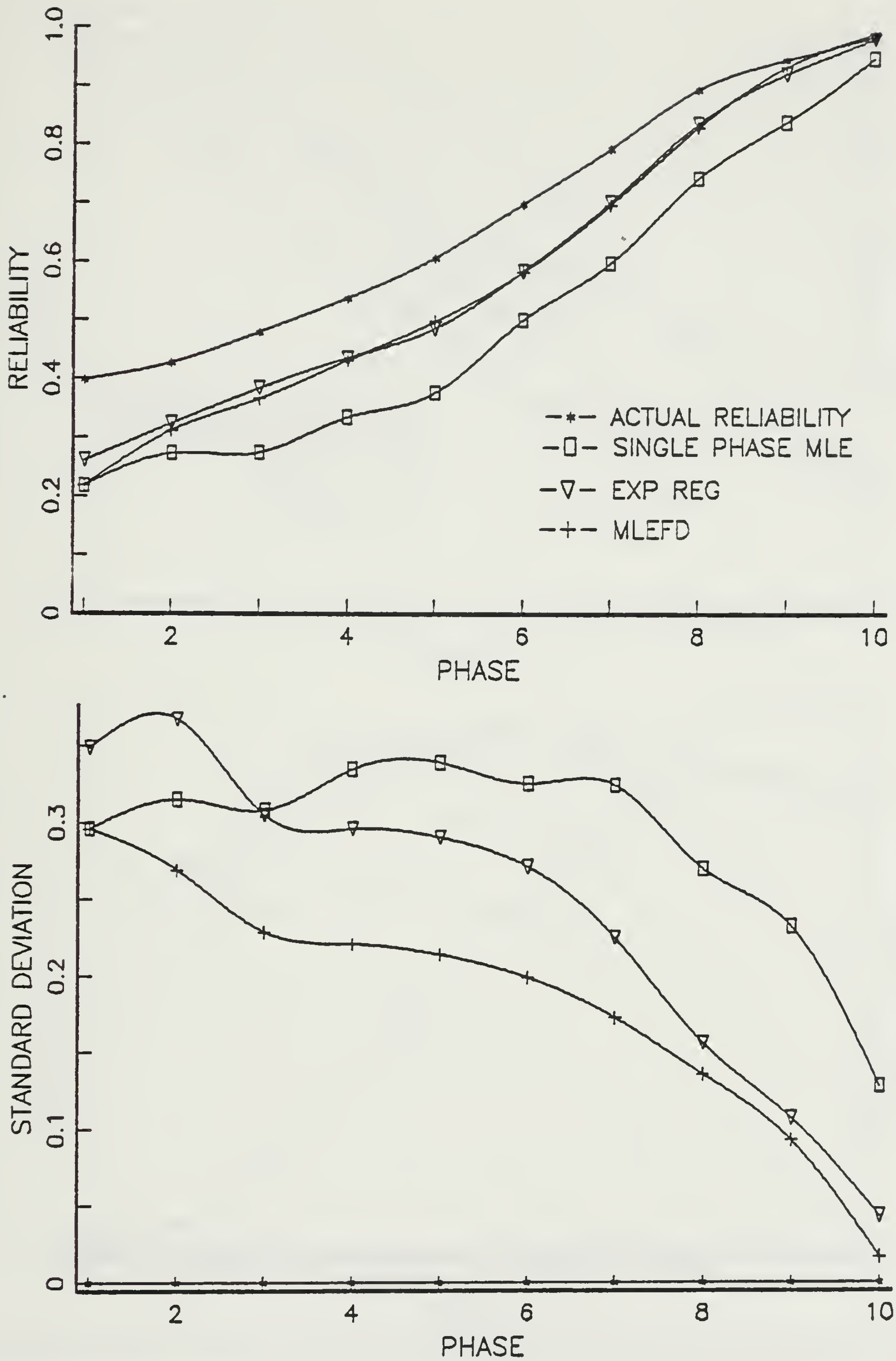

Figure 32. Pattern I, F = .50, I = 6

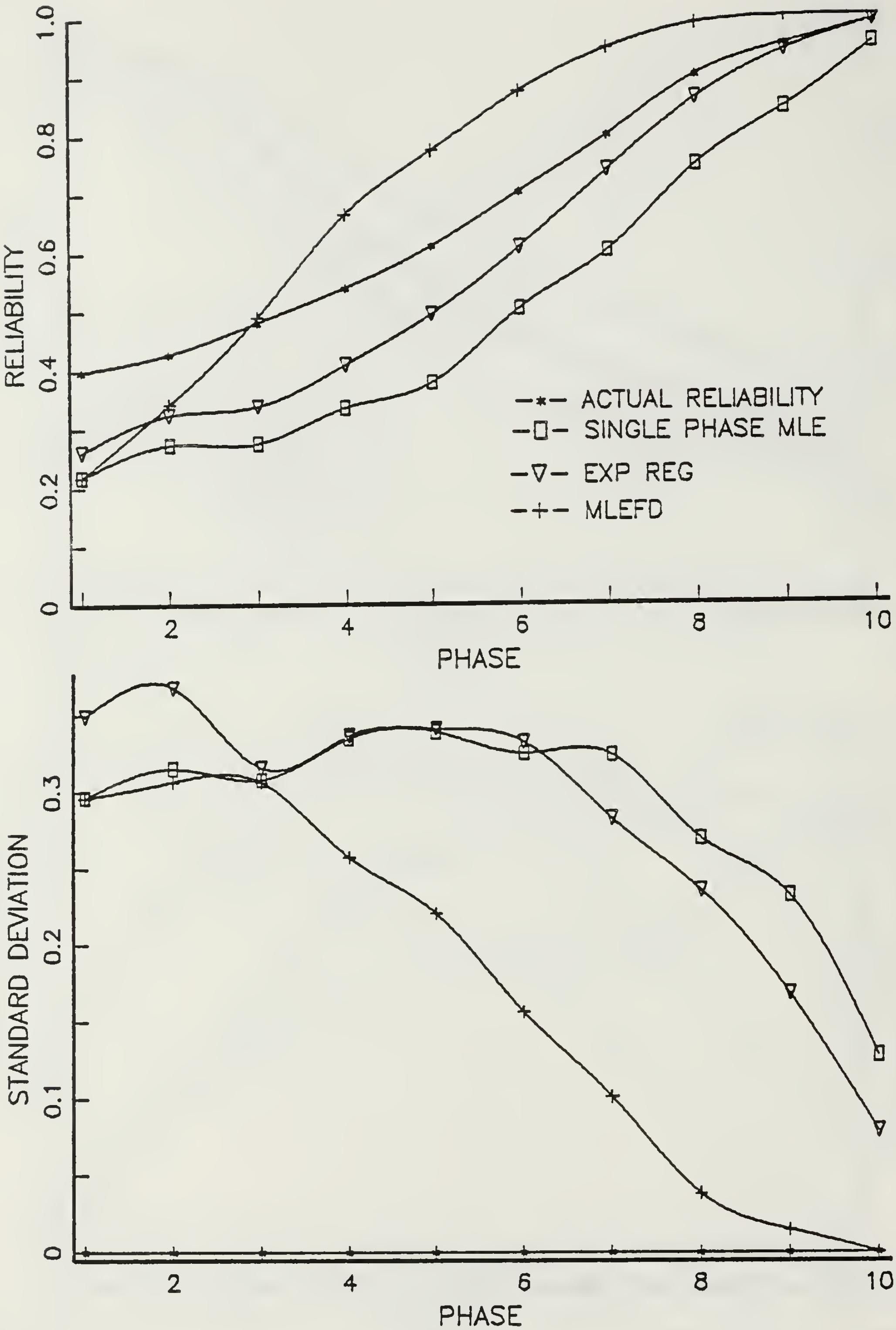

Figure 33. Pattern I, F = .75, I = 3

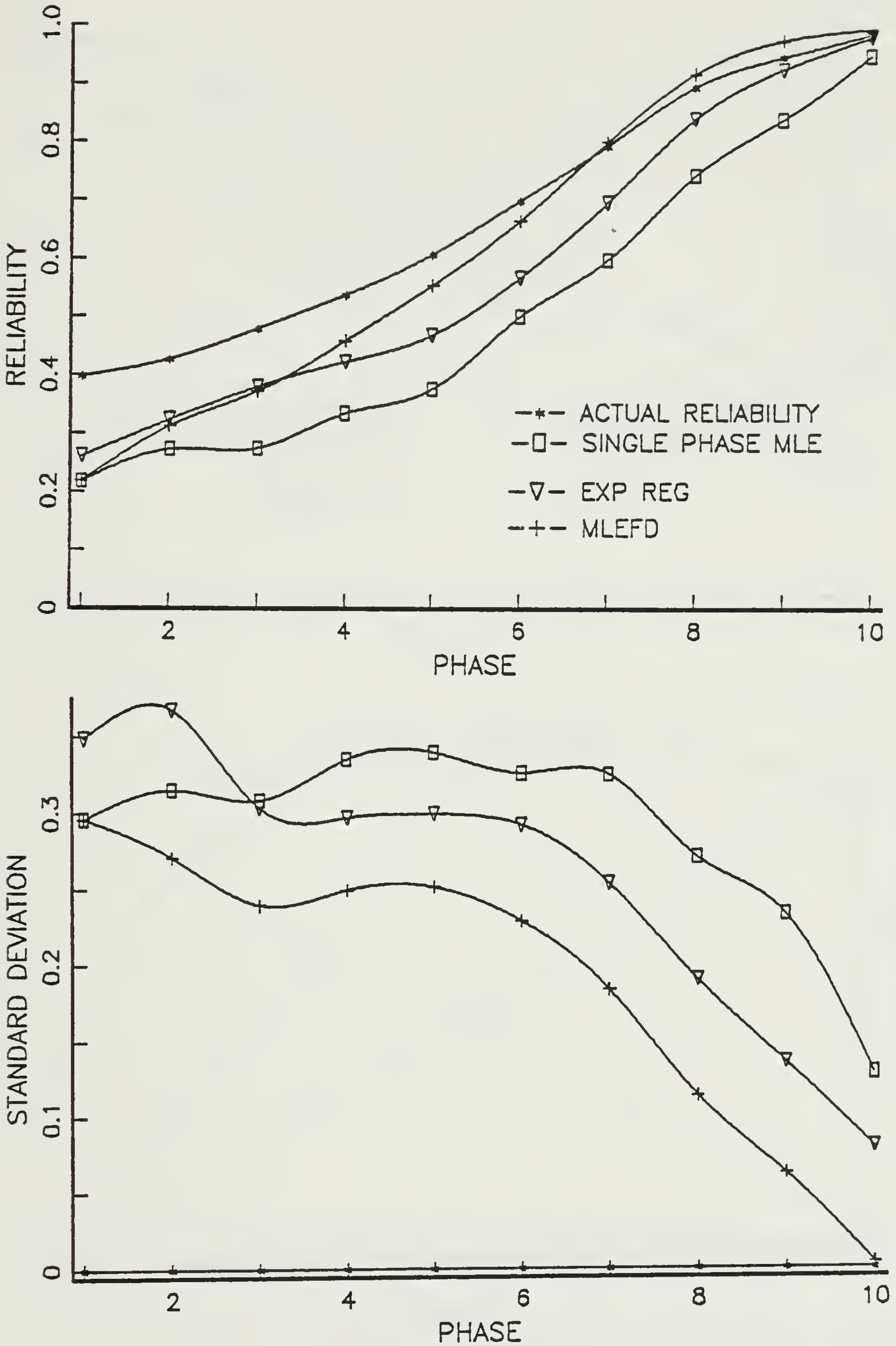

Figure 34. Pattern I, F = .75, I = 6

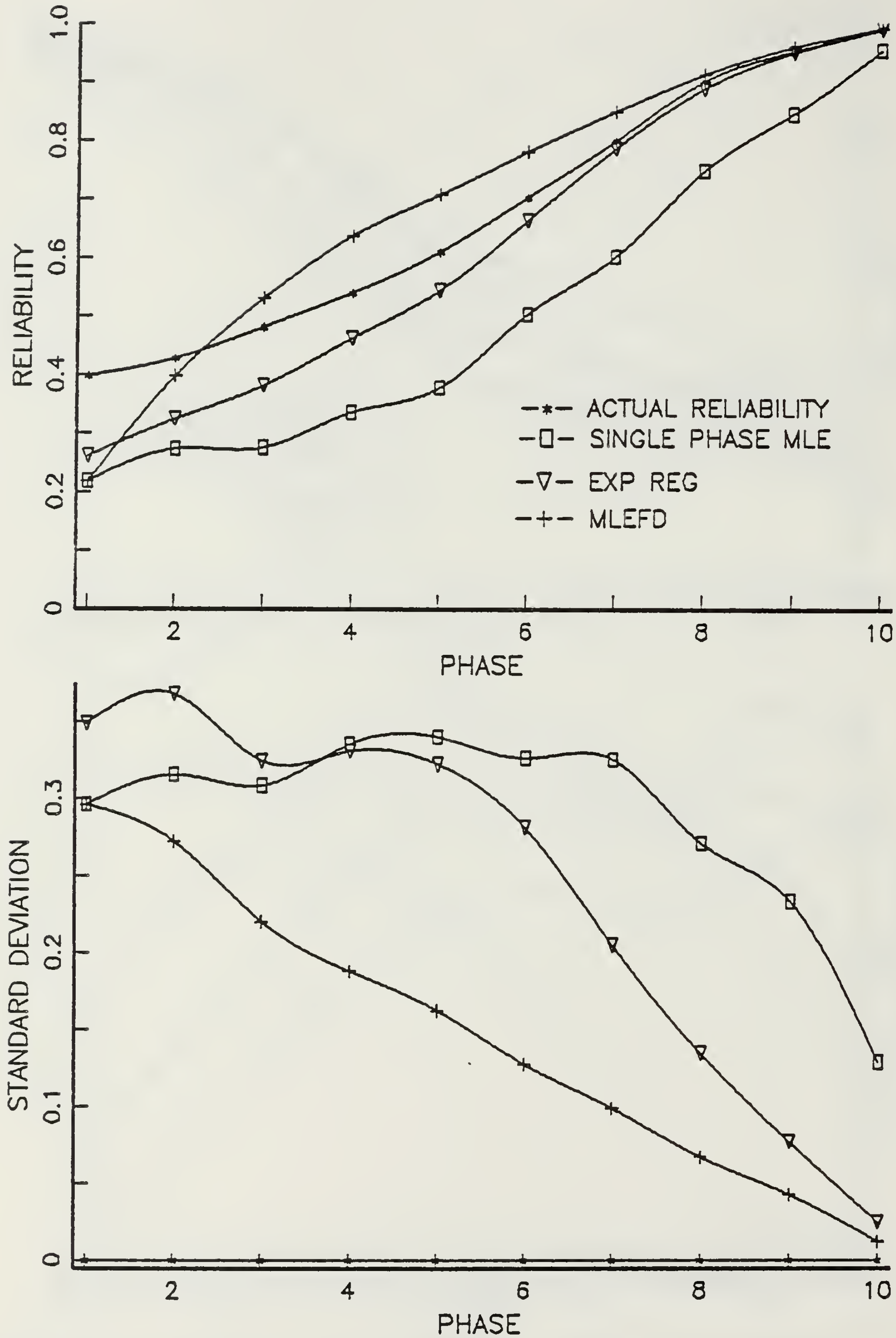

Figure 35. Pattern I, Lloyd, CI = .8

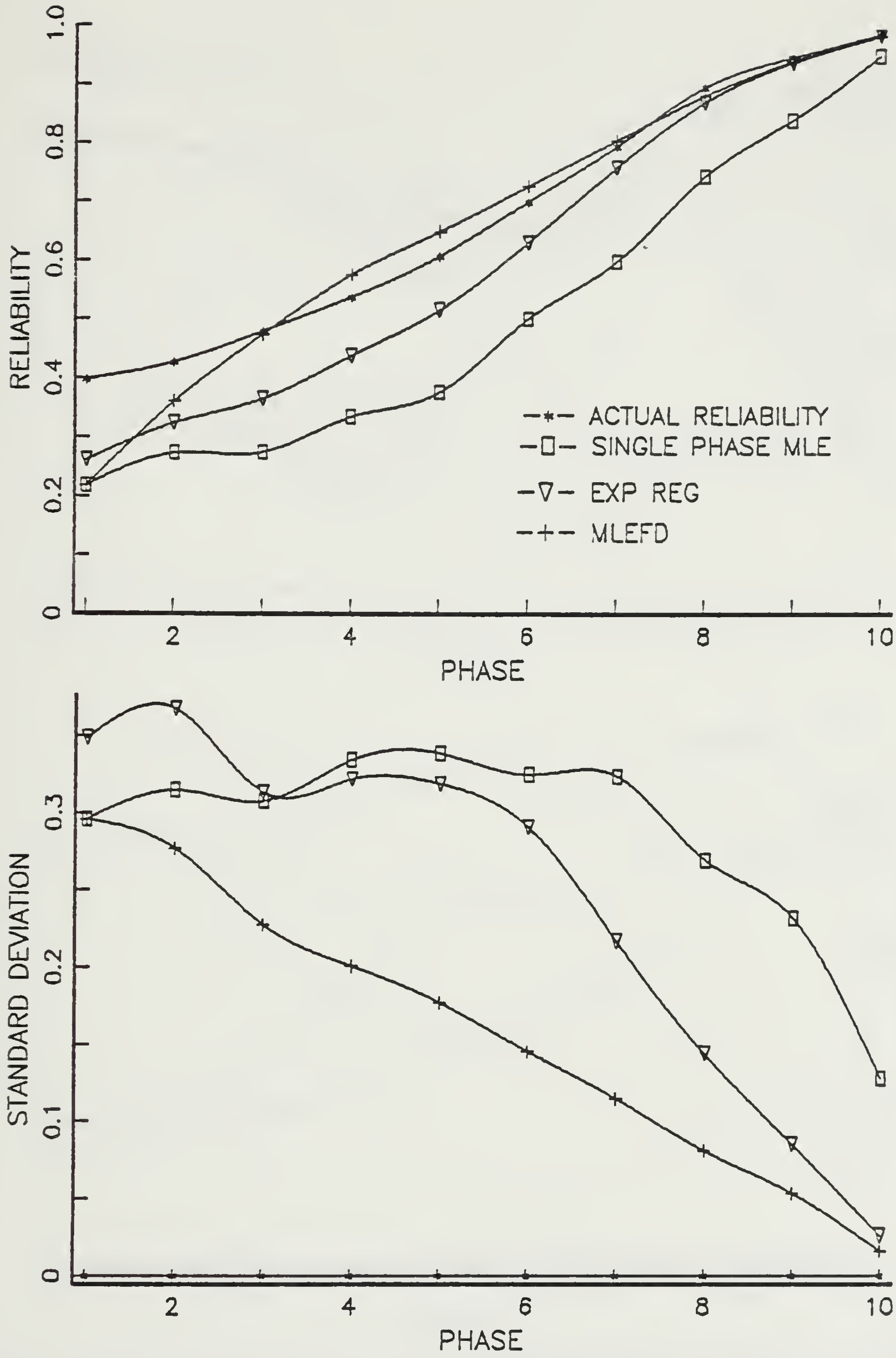

Figure 36. Pattern I, Lloyd, CI = .9

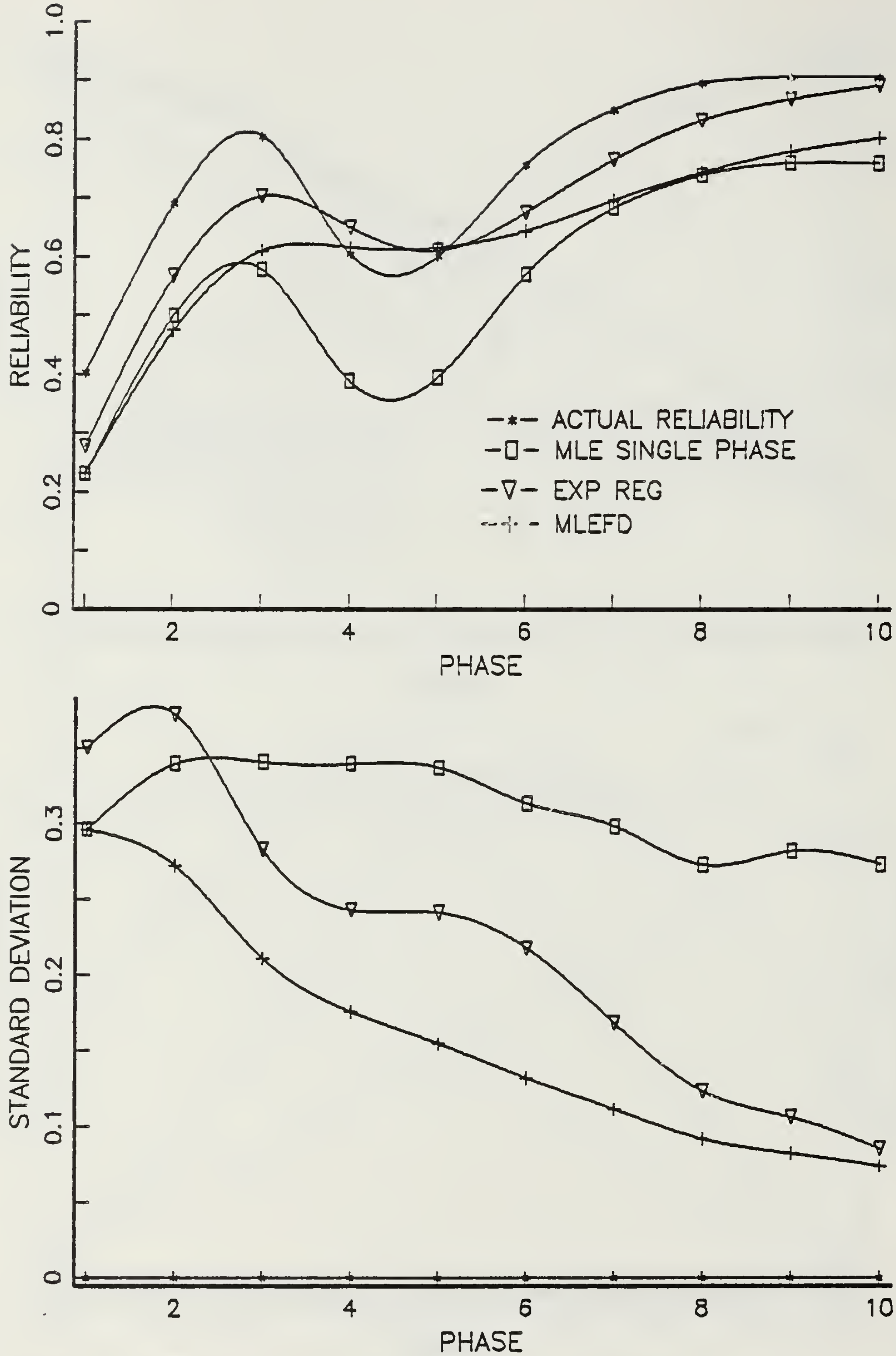

Figure 37. Pattern II, No Discounting

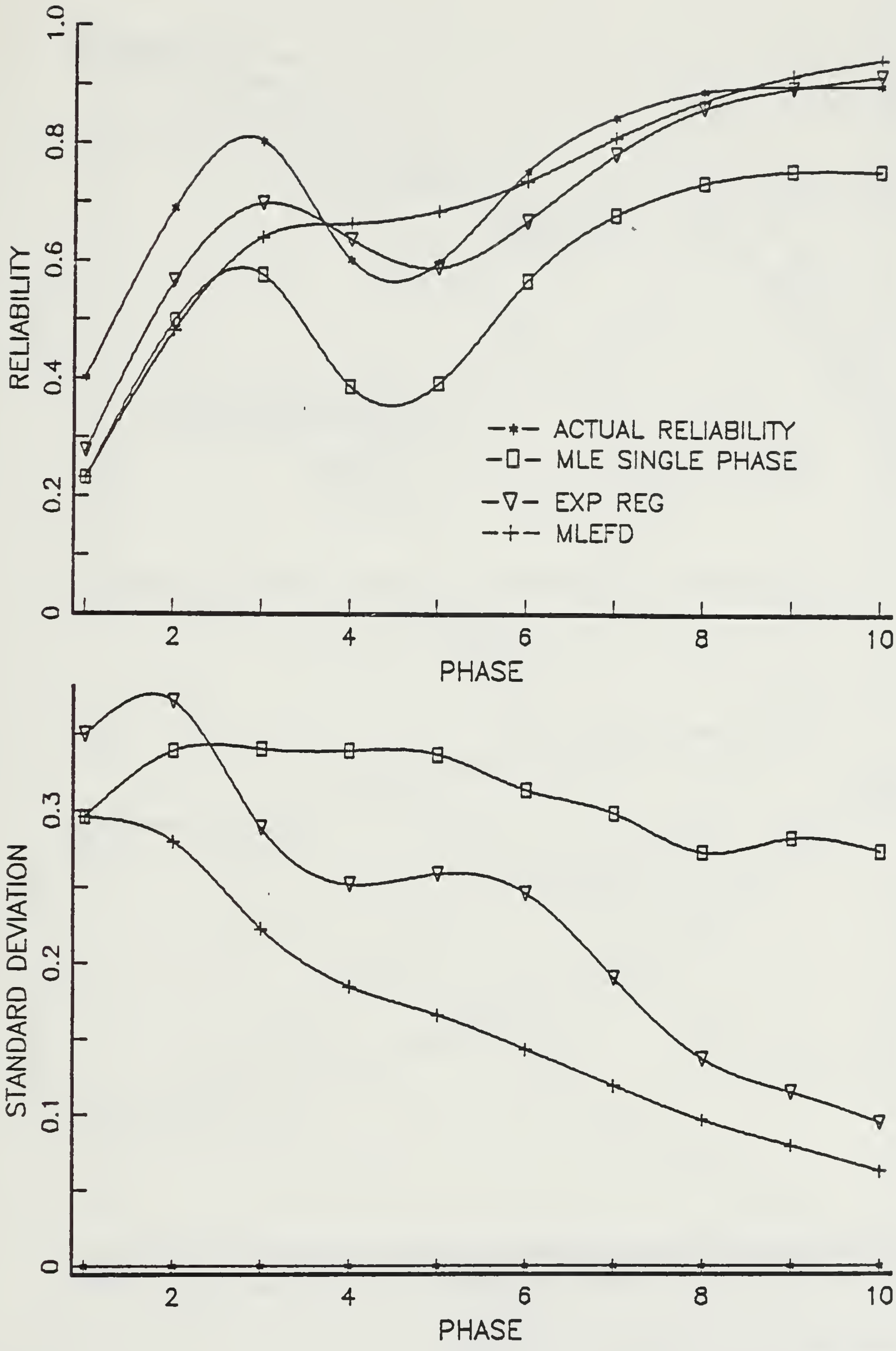

Figure 38. Pattern II, F = .25, I = 3

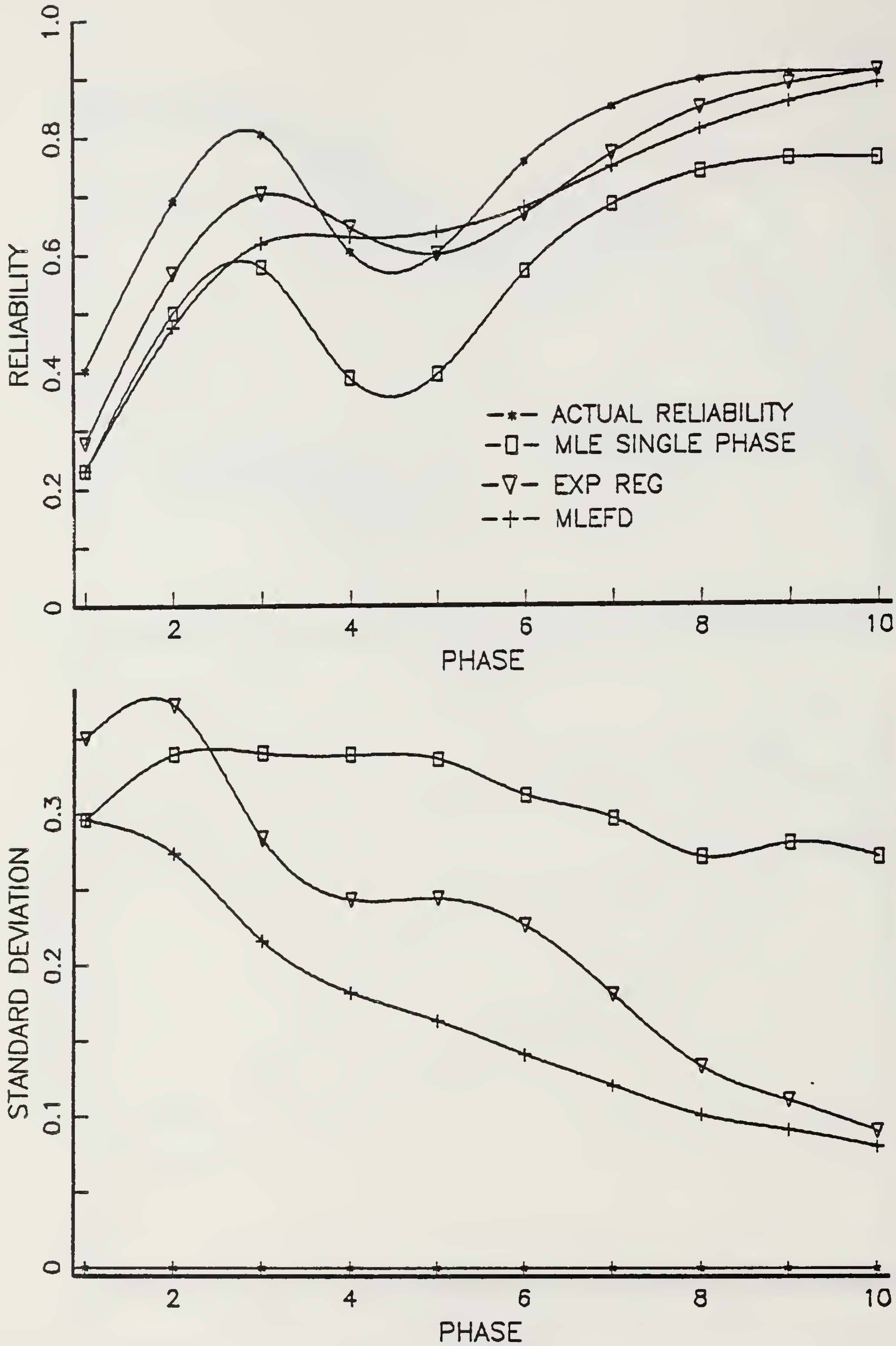

Figure 39. Pattern II, F = .25, I = 6

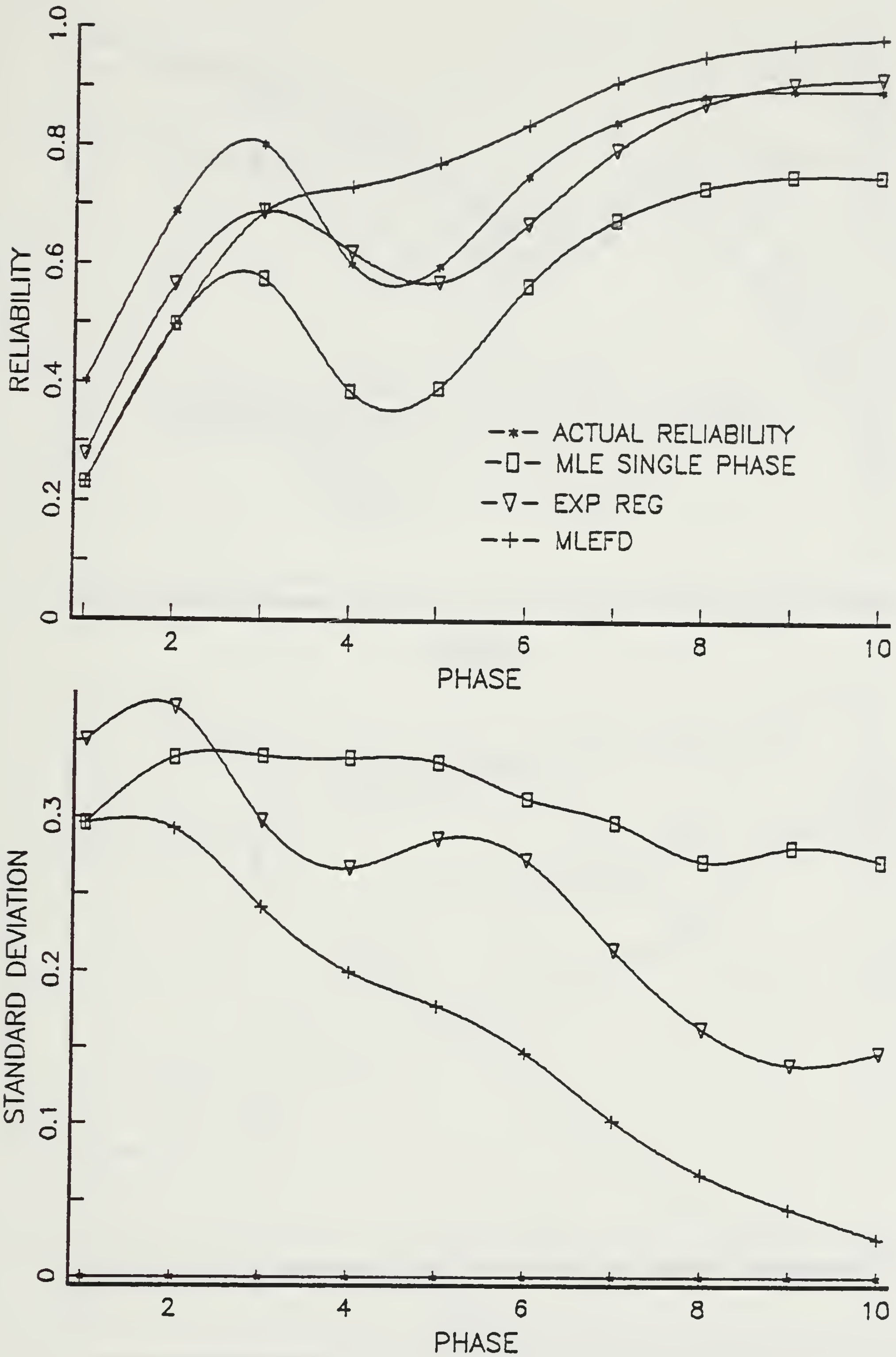

Figure 40. Pattern II, F = .50, I = 3

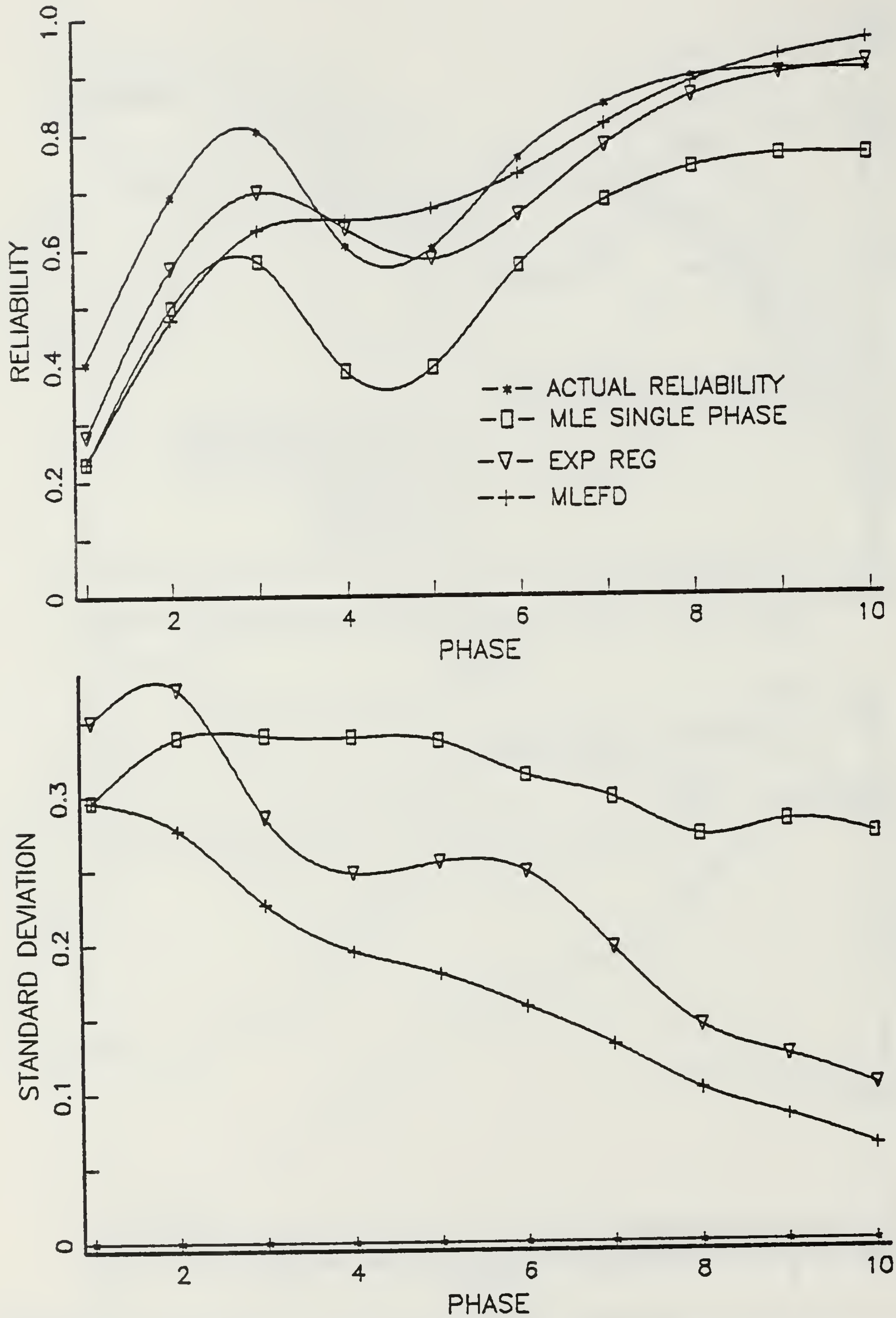

Figure 41. Pattern II, F = .50, I = 6

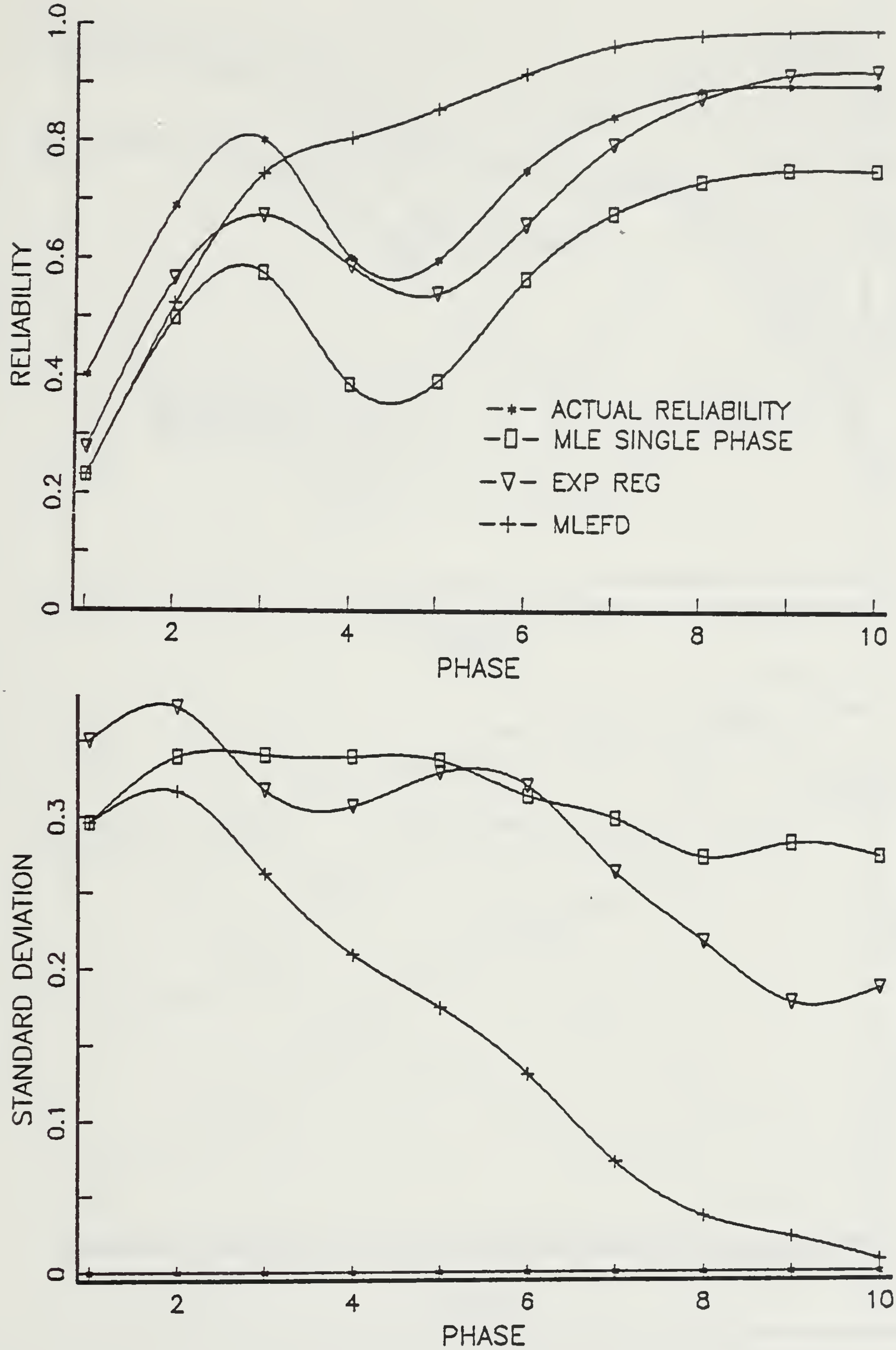

Figure 42. Pattern II, F = .75, I = 3

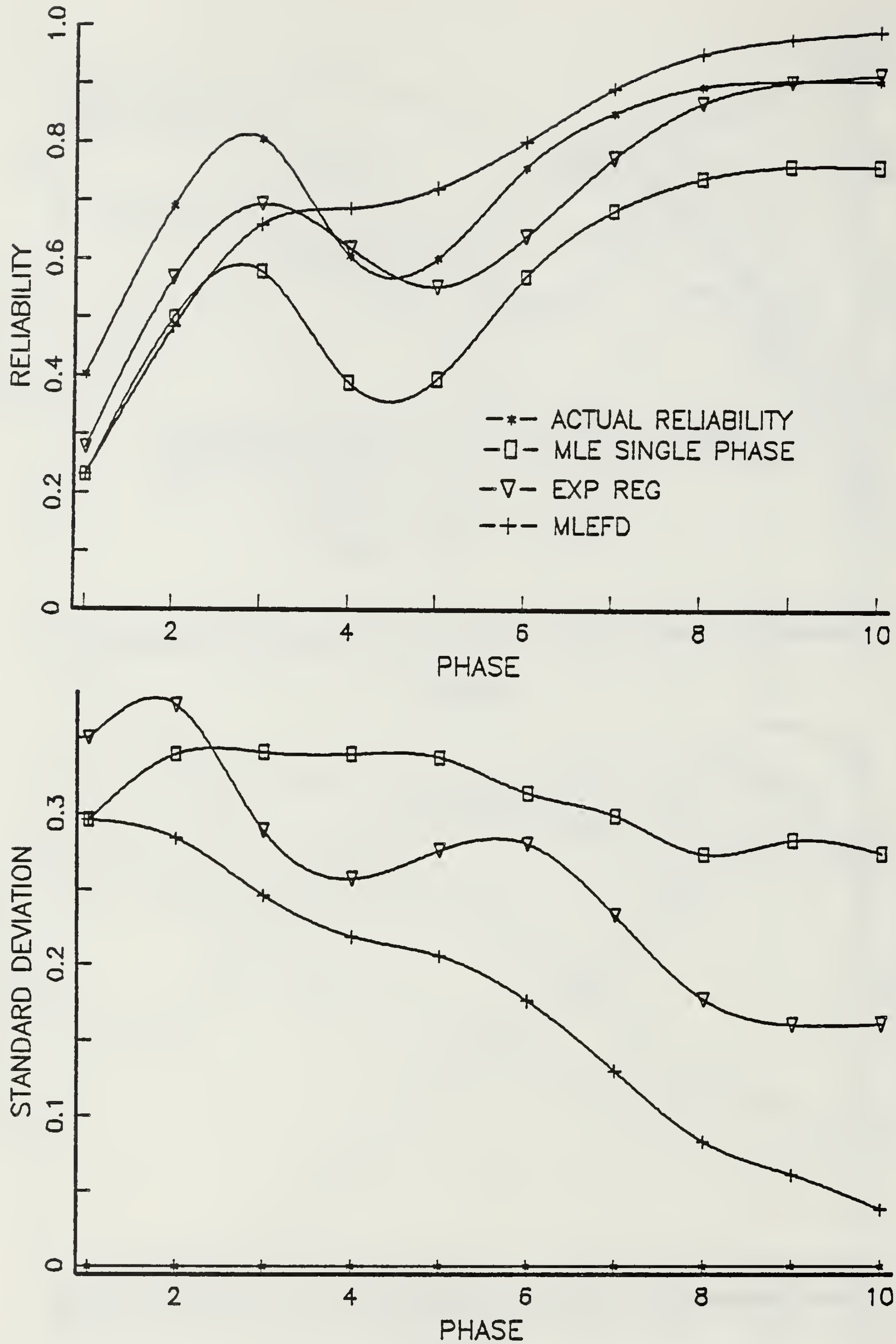

Figure 43. Pattern II, F = .75, I = 6

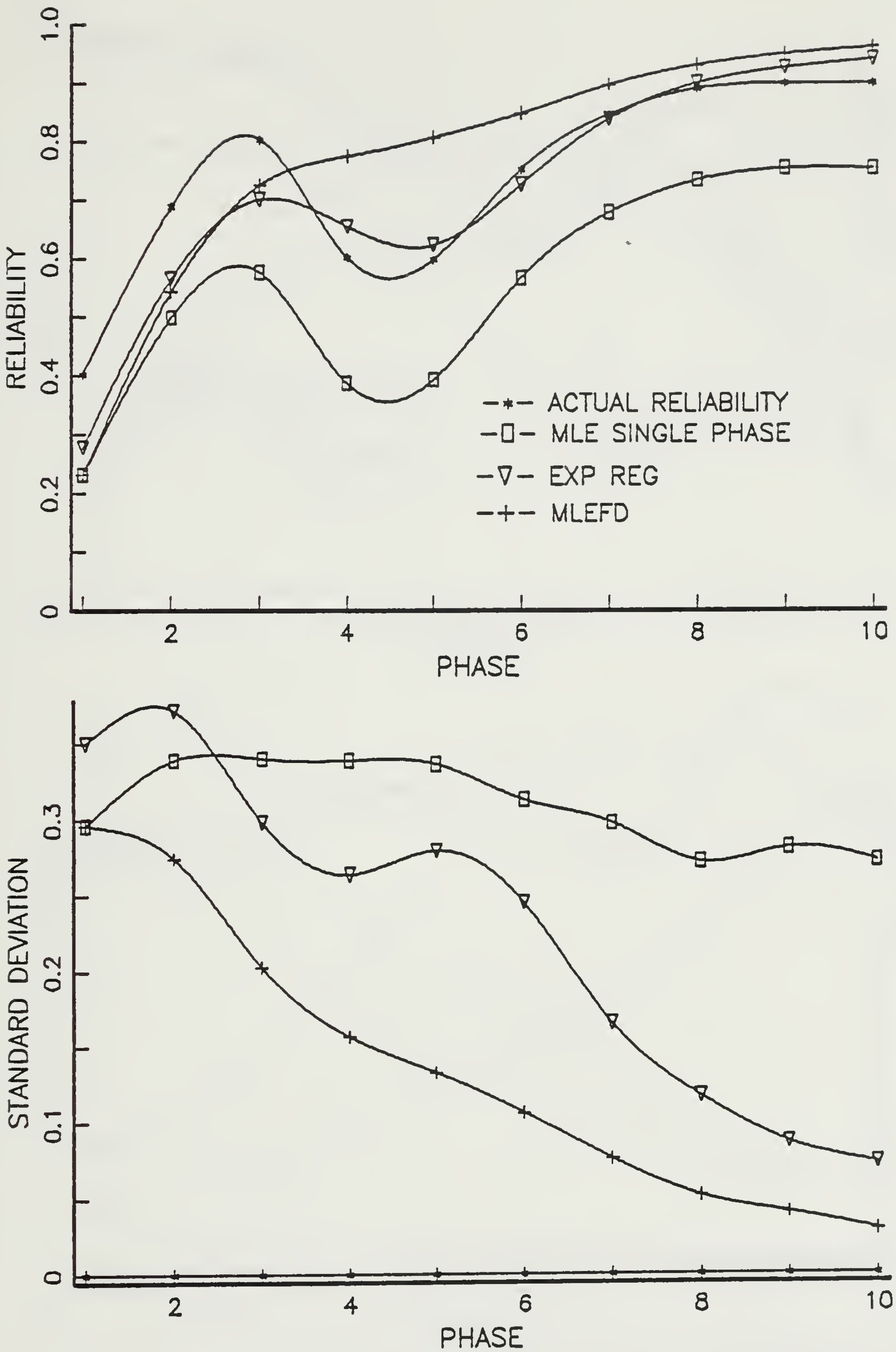

Figure 44. Pattern II, Lloyd, CI = .8

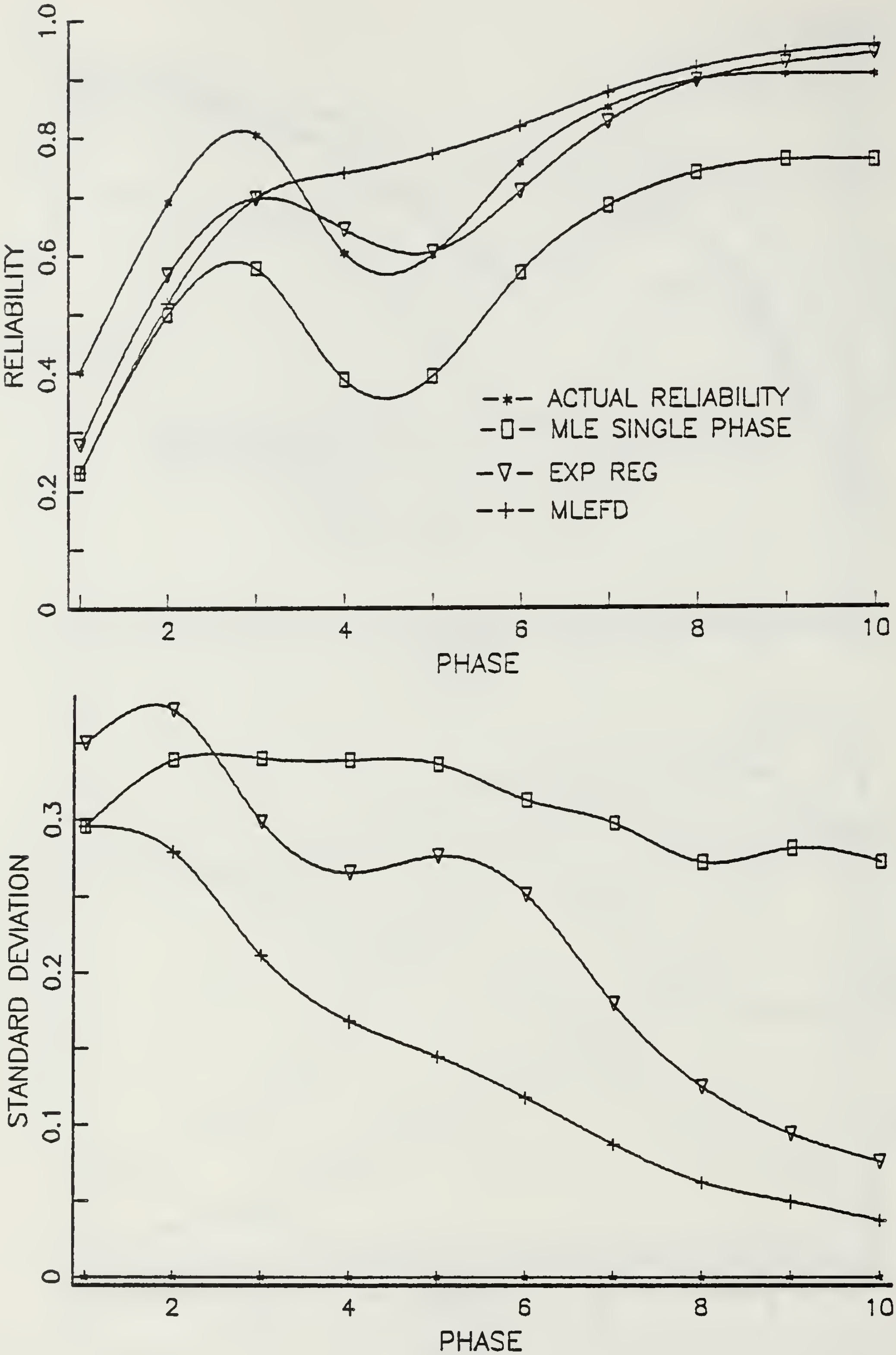

Figure 45. Pattern II, Lloyd, CI = .9

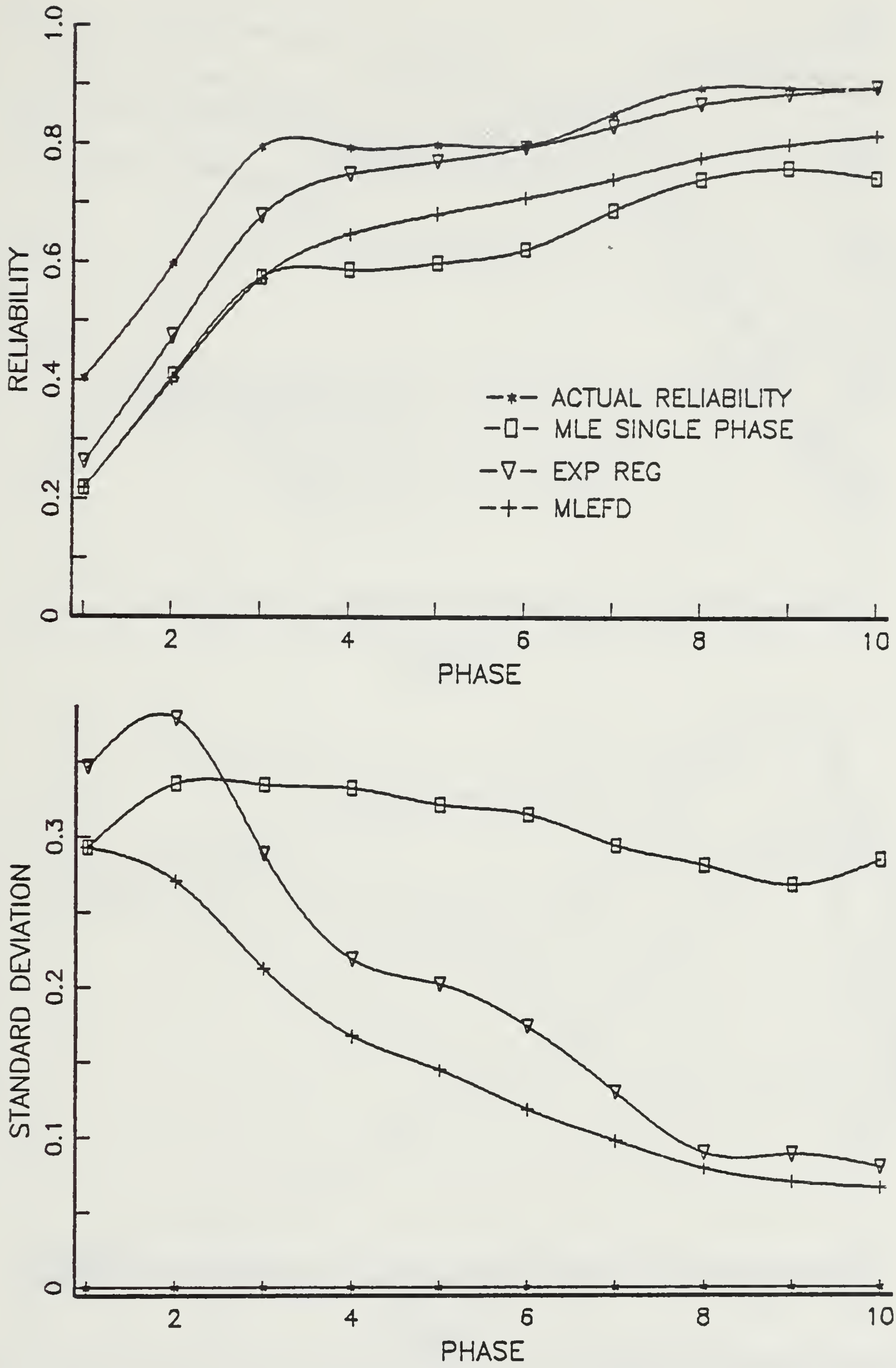

Figure 46. Pattern III, No Discounting

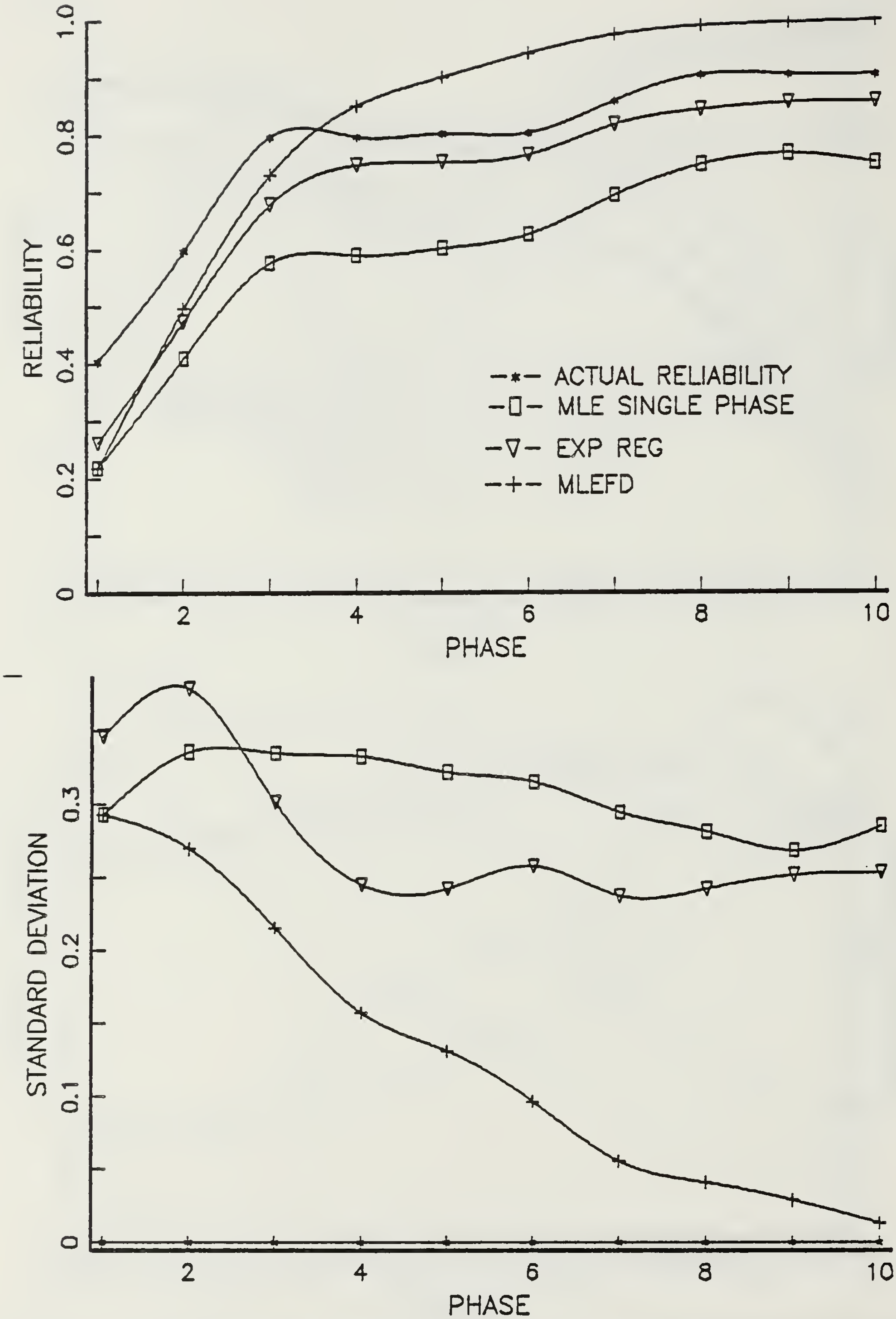

Figure 47. Pattern III, F = .25, I = 1

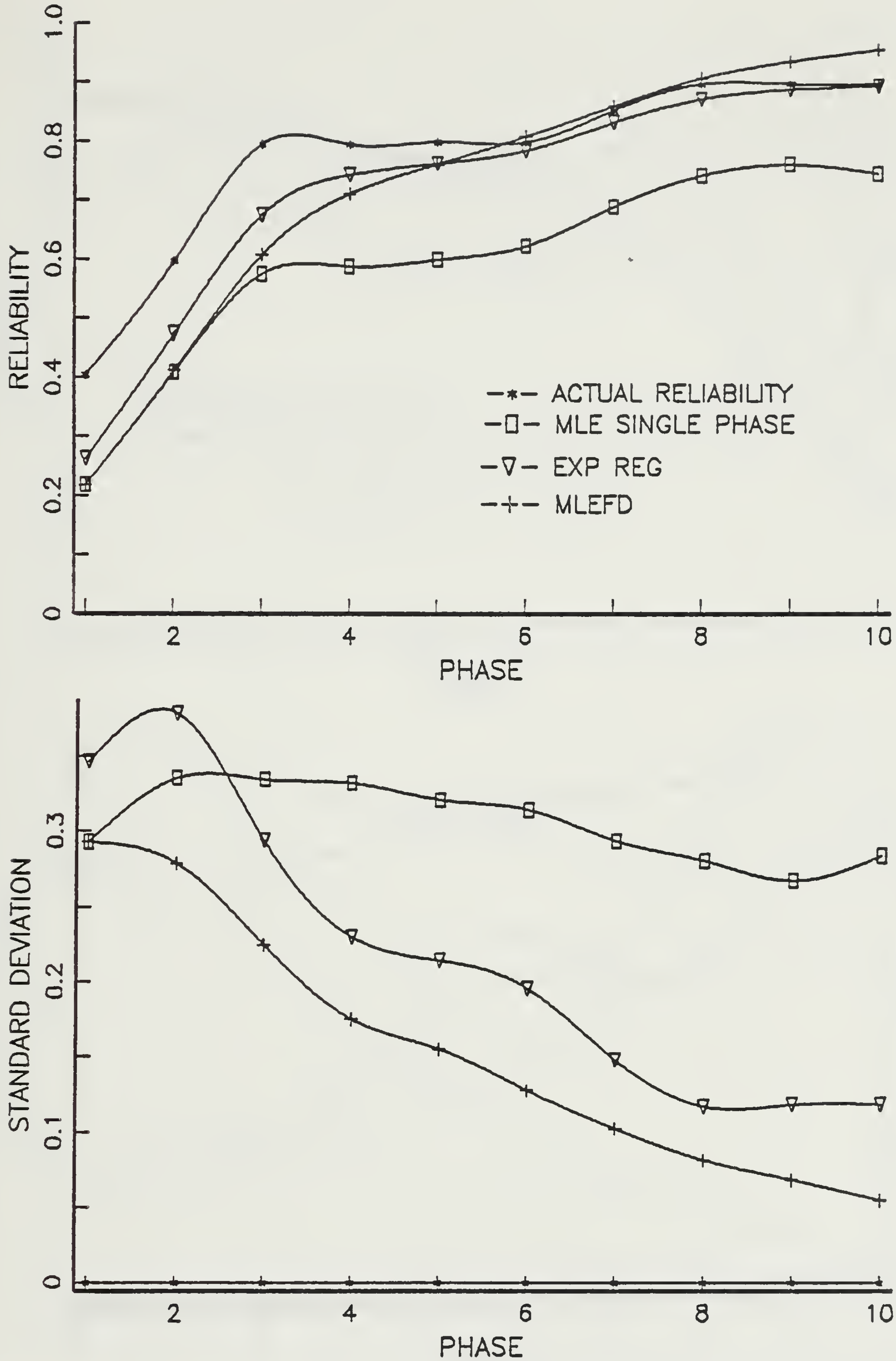

Figure 48. Pattern III, F = .25, I = 3

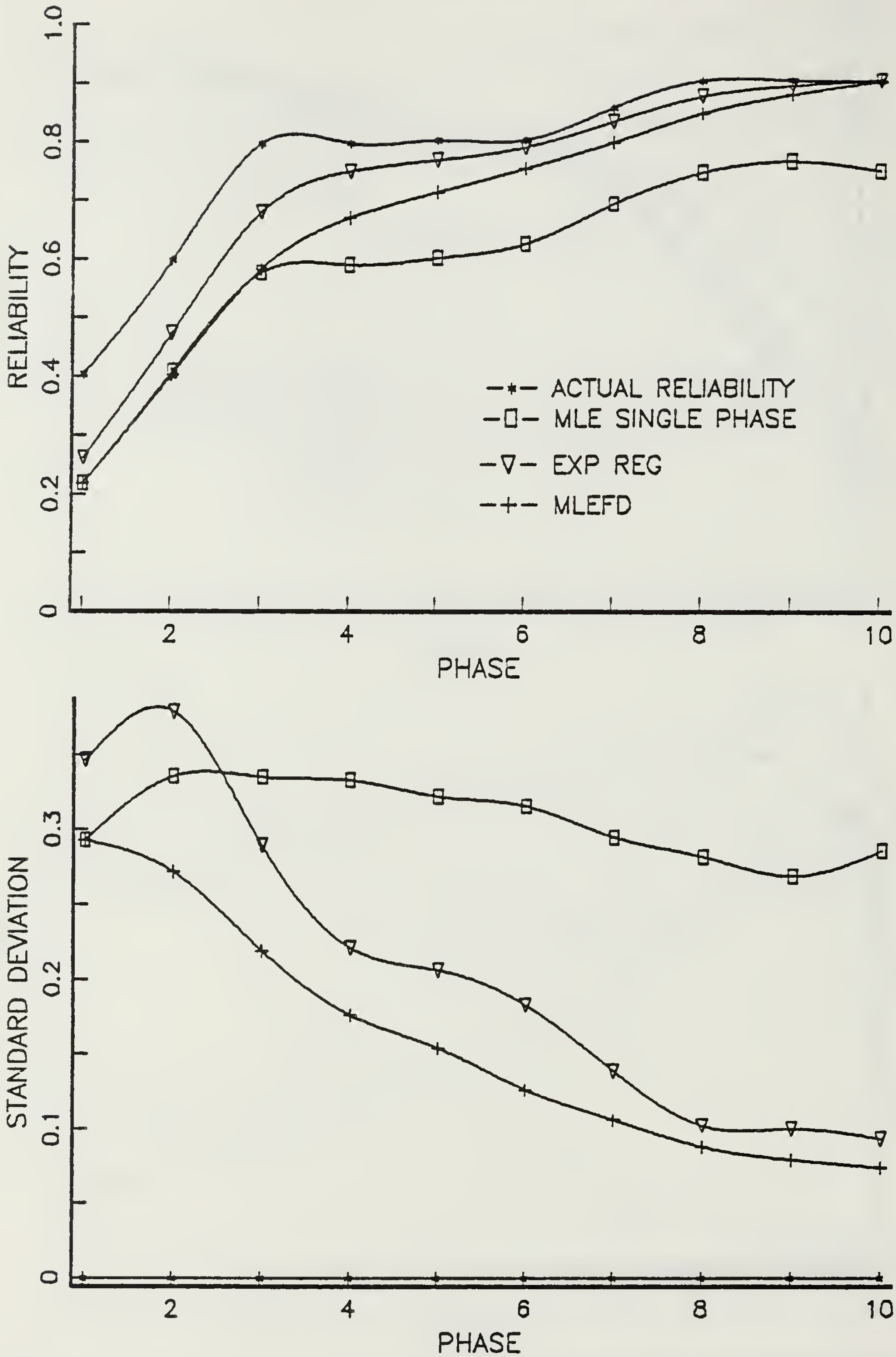

Figure 49. Pattern III, F = .25, I = 6

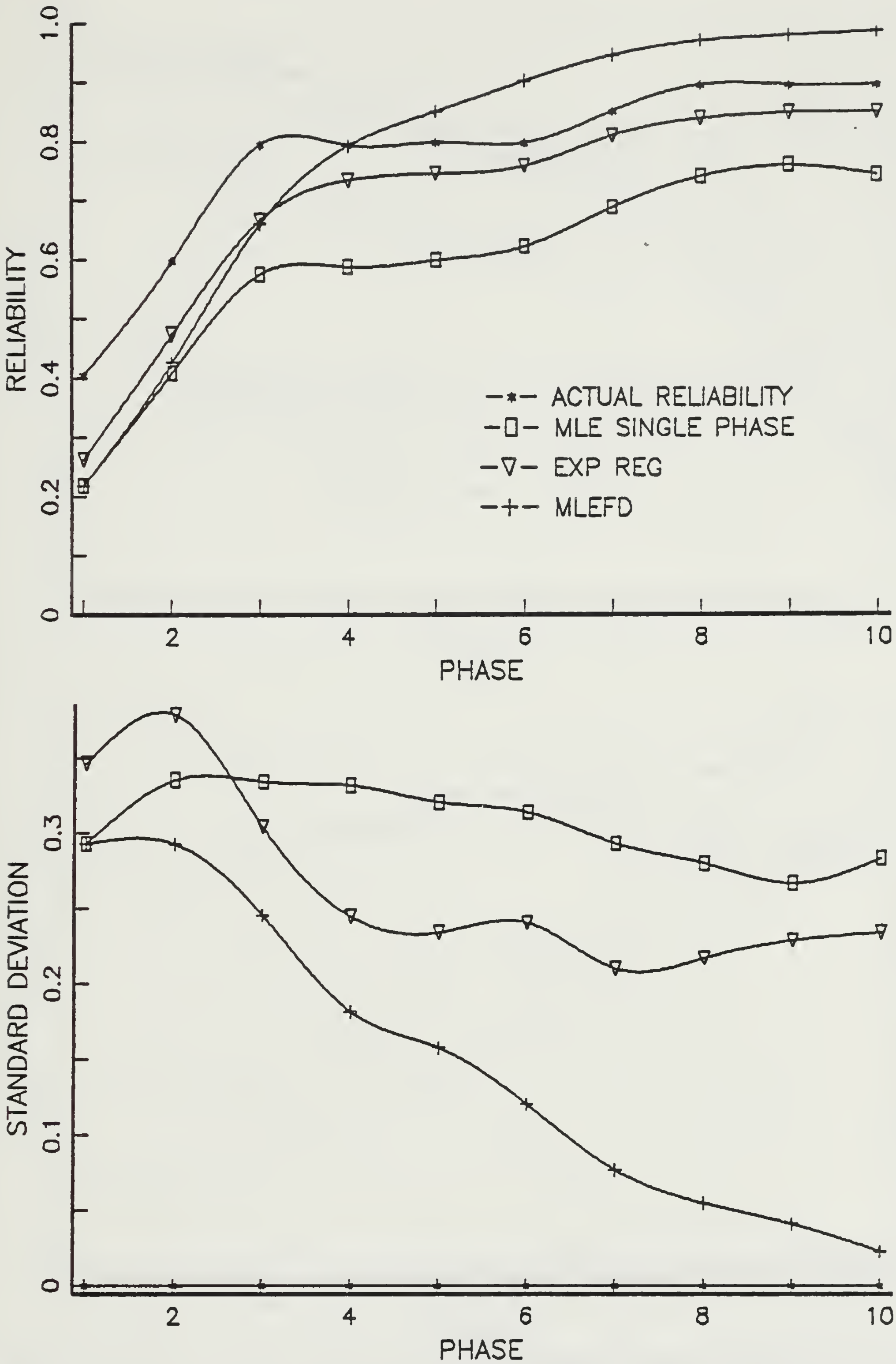

Figure 50. Pattern III, F = .50, I = 3

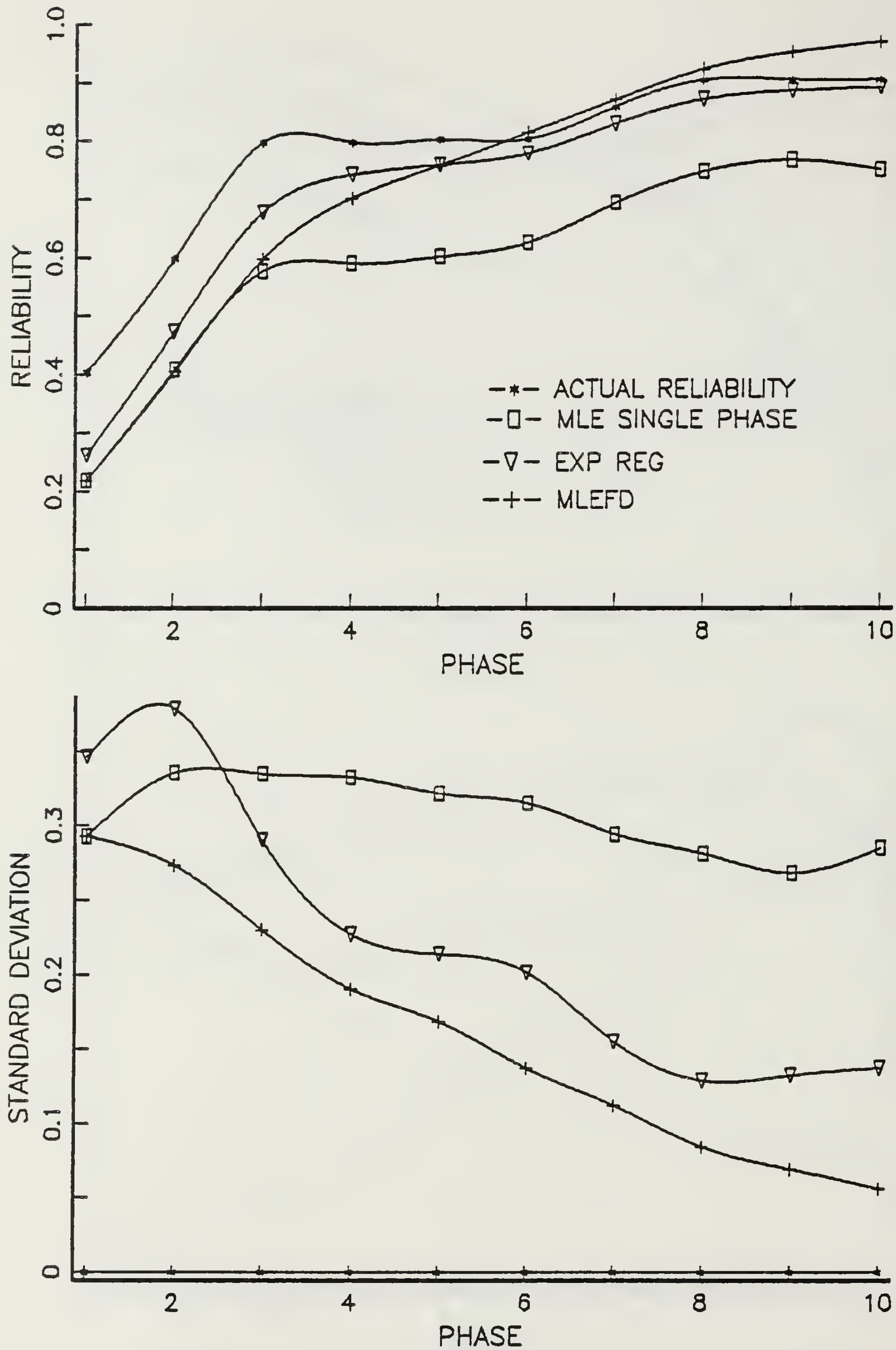

Figure 51. Pattern III, F = .50, I = 6

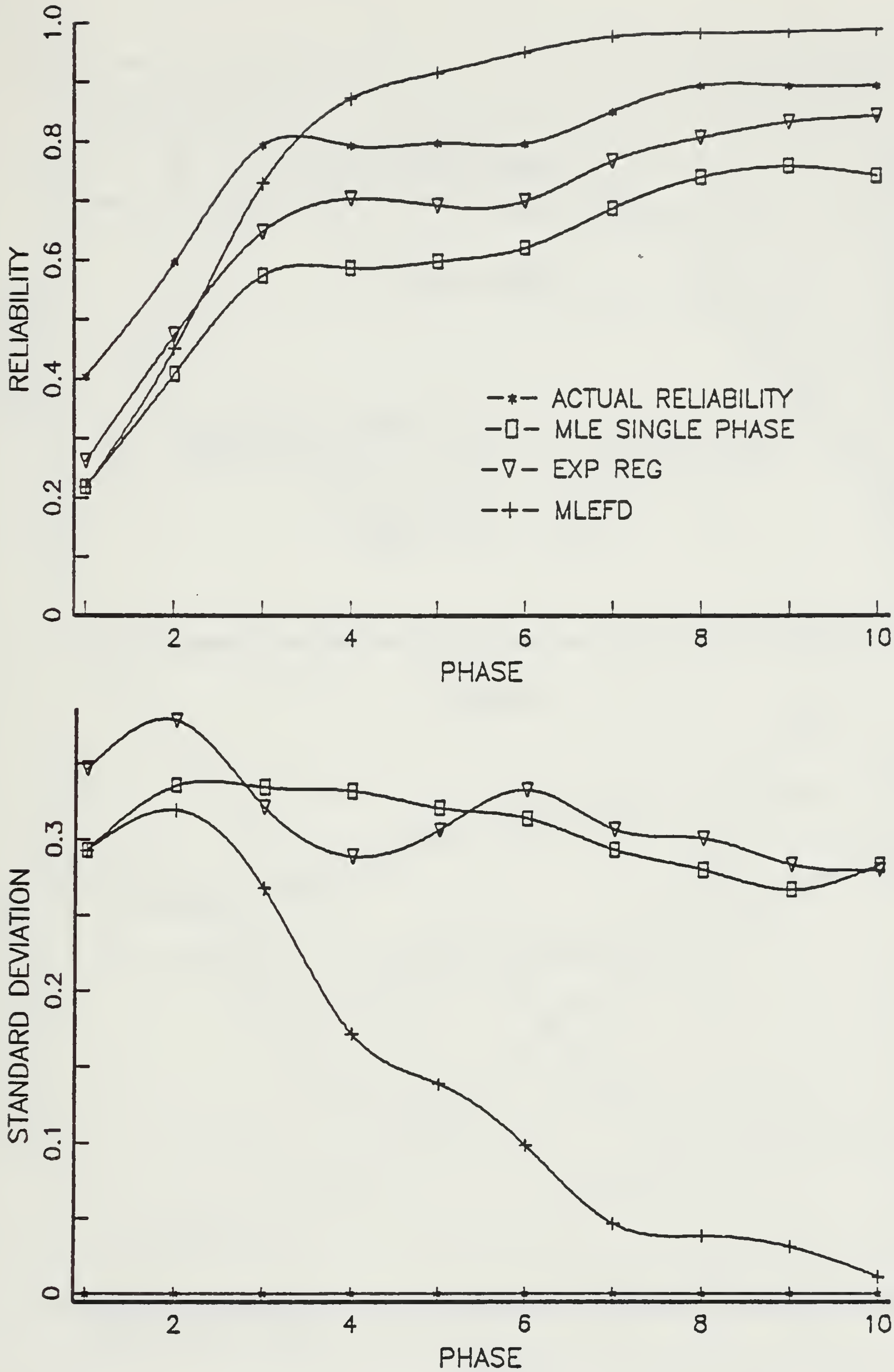

Figure 52. Pattern III, F = .75, I = 3

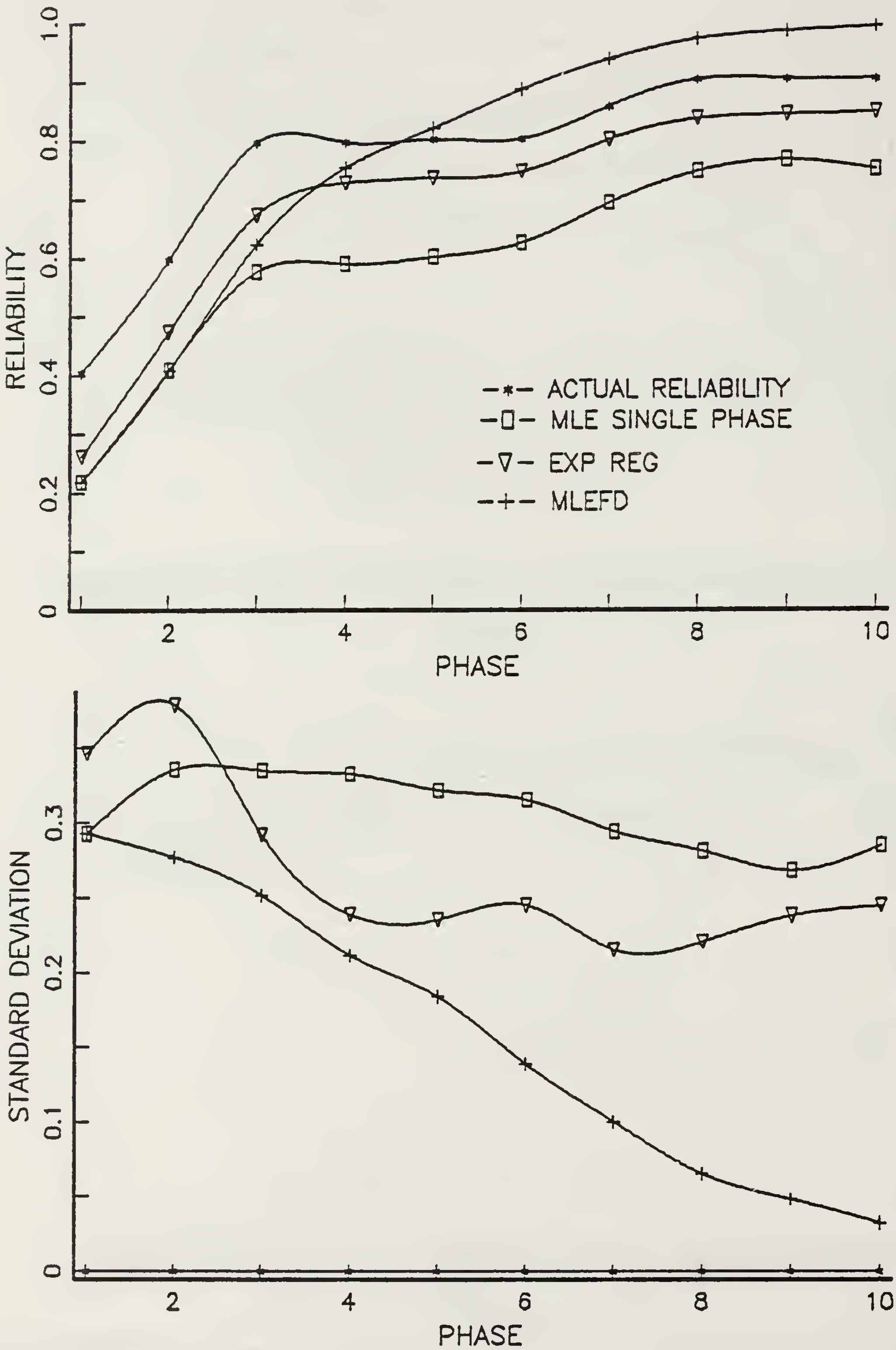

Figure 53. Pattern III, F = .75, I = 6

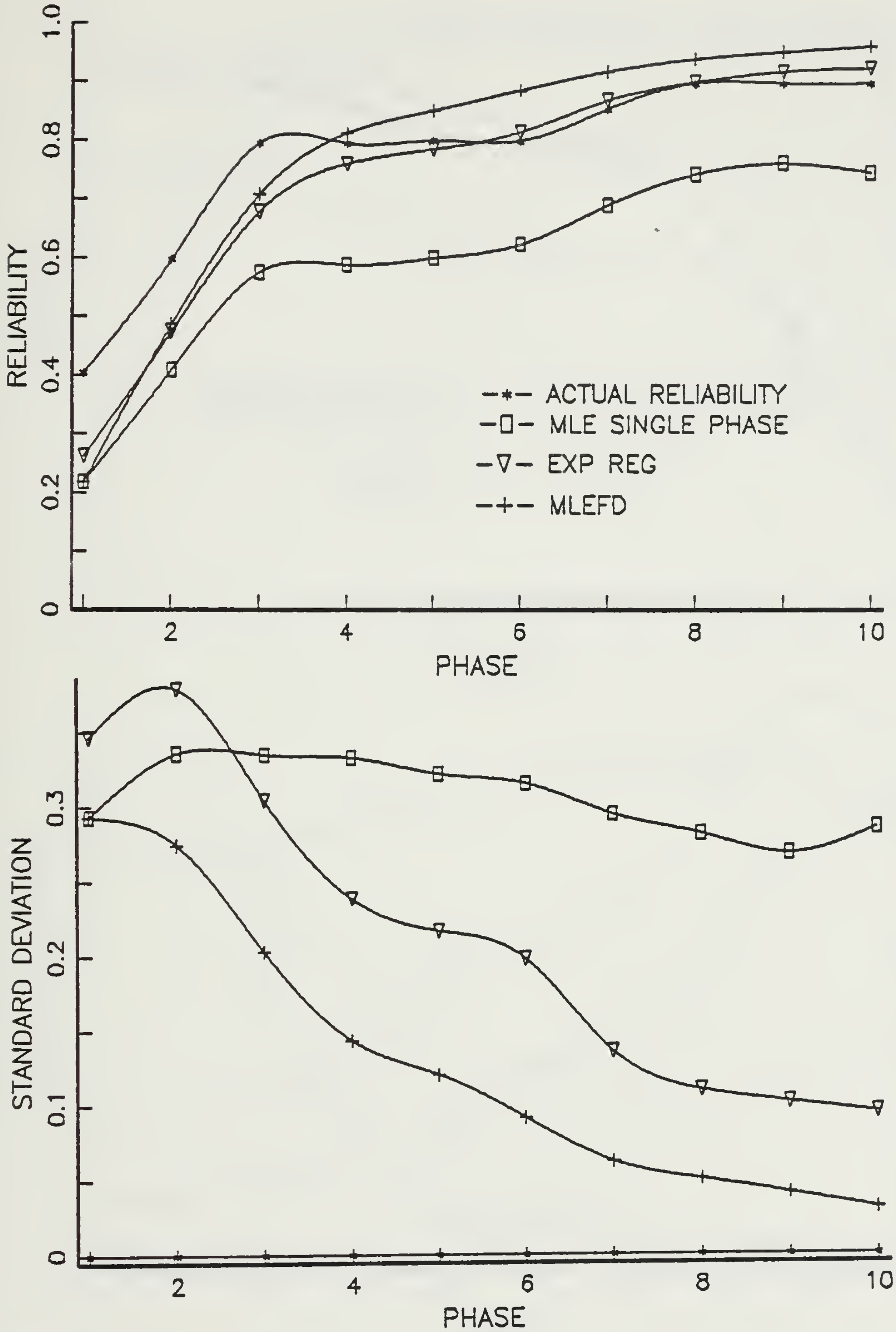

Figure 54. Pattern III, Lloyd, CI = .8

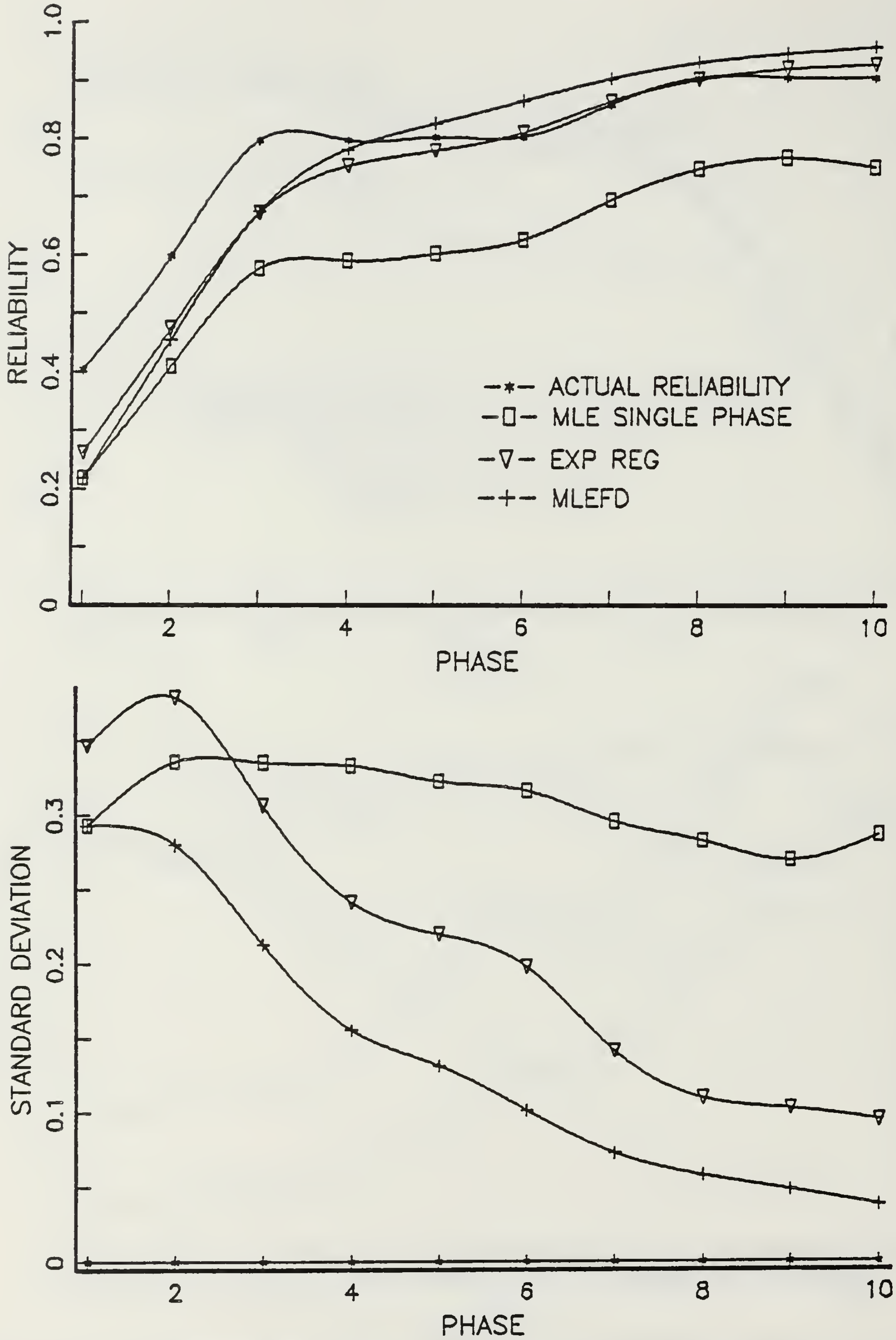

Figure 55. Pattern III, Lloyd, CI = .9

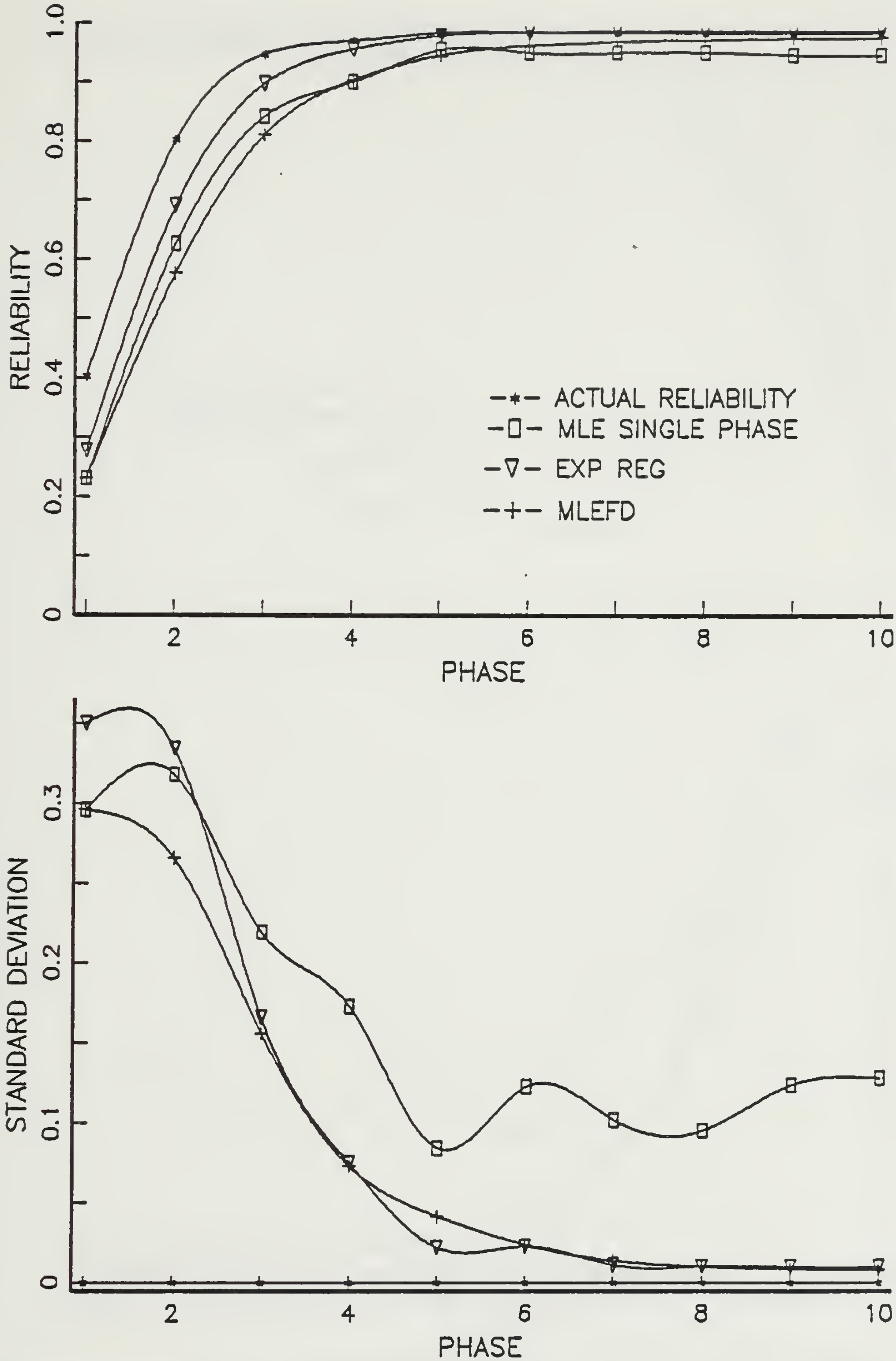

Figure 56. Pattern IV, No Discounting

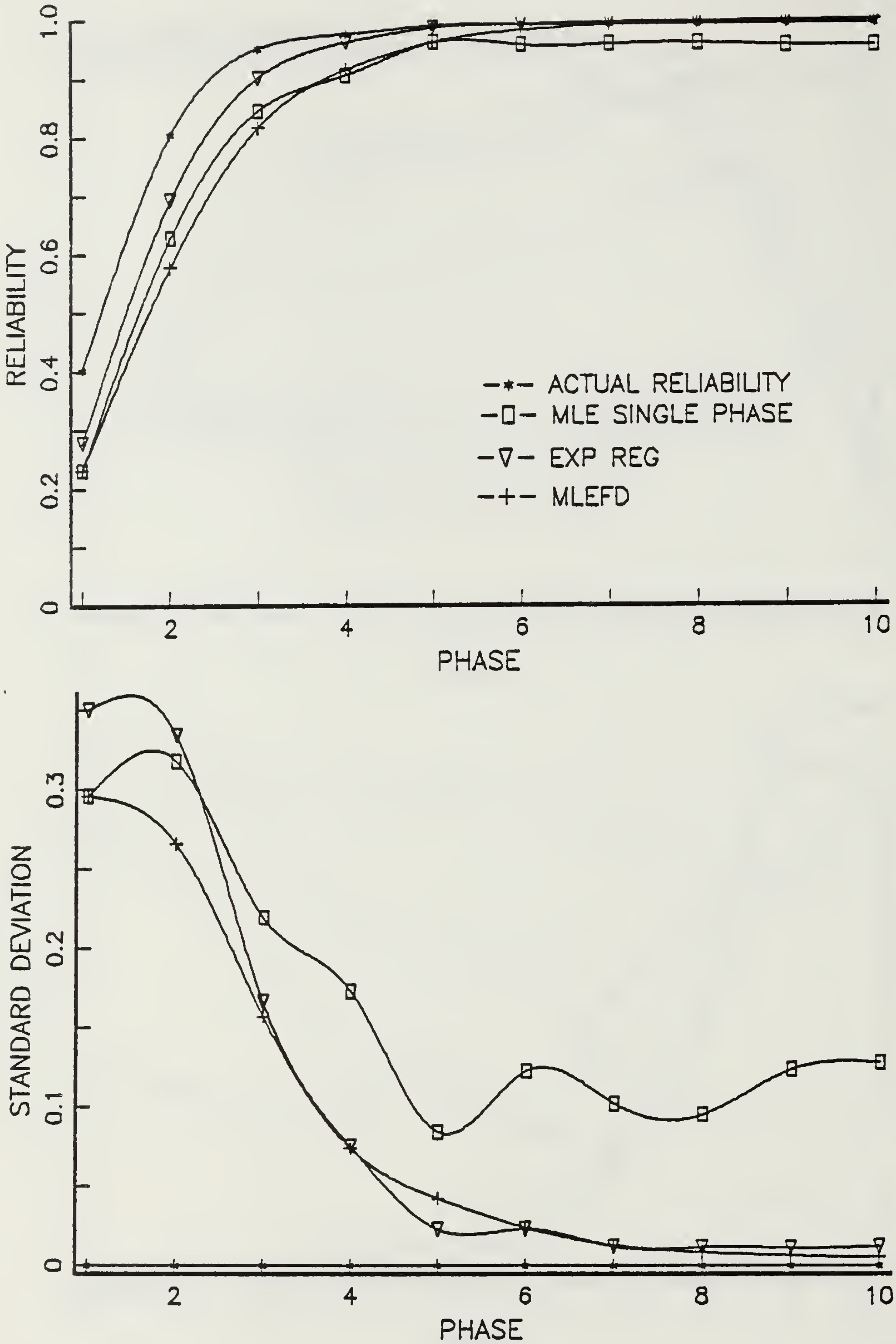

Figure 57. Pattern IV, F = .10 I = 10

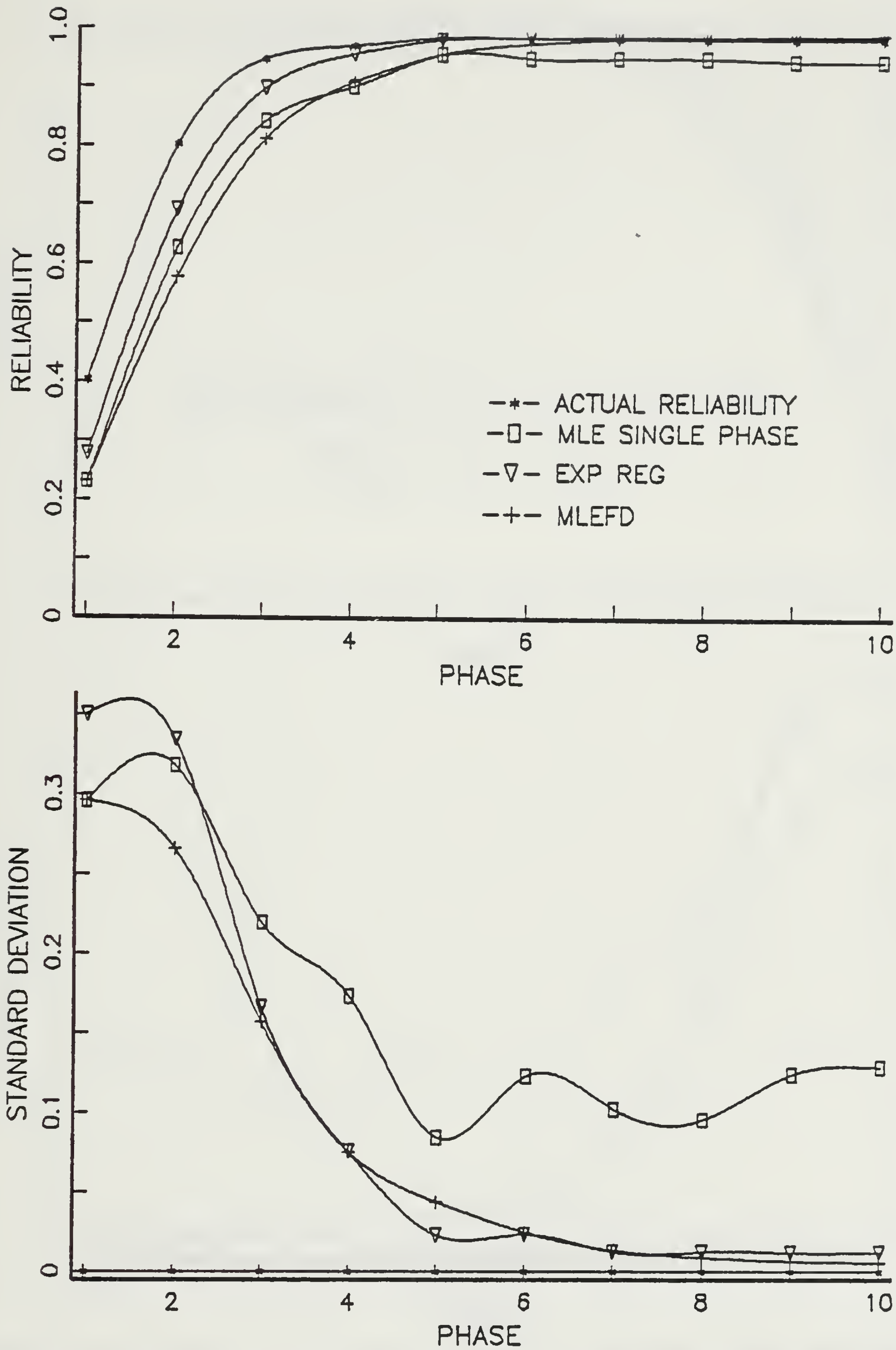

Figure 58. Pattern IV, F = .25, I = 25

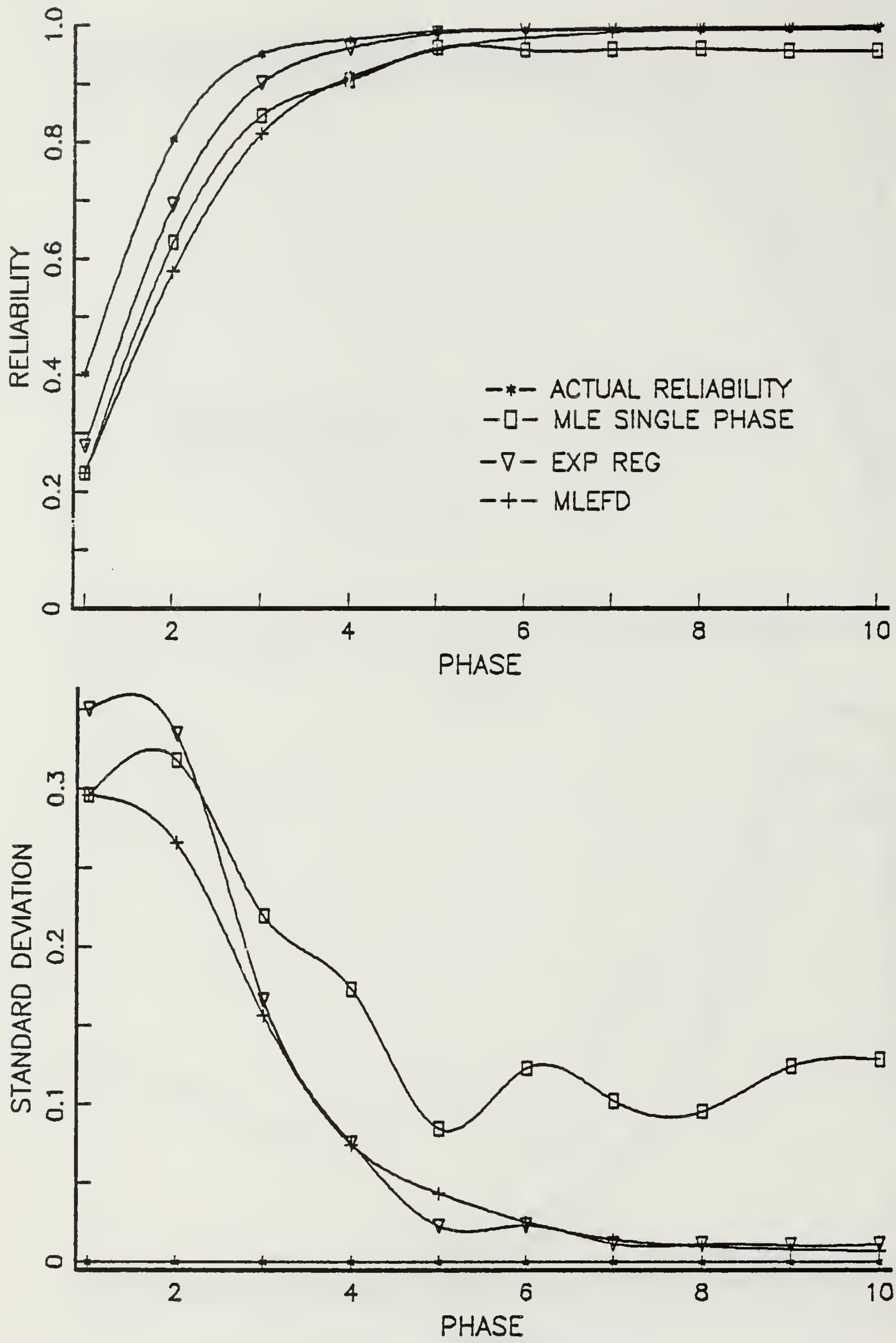

Figure 59. Pattern IV, F = .25, I = 35

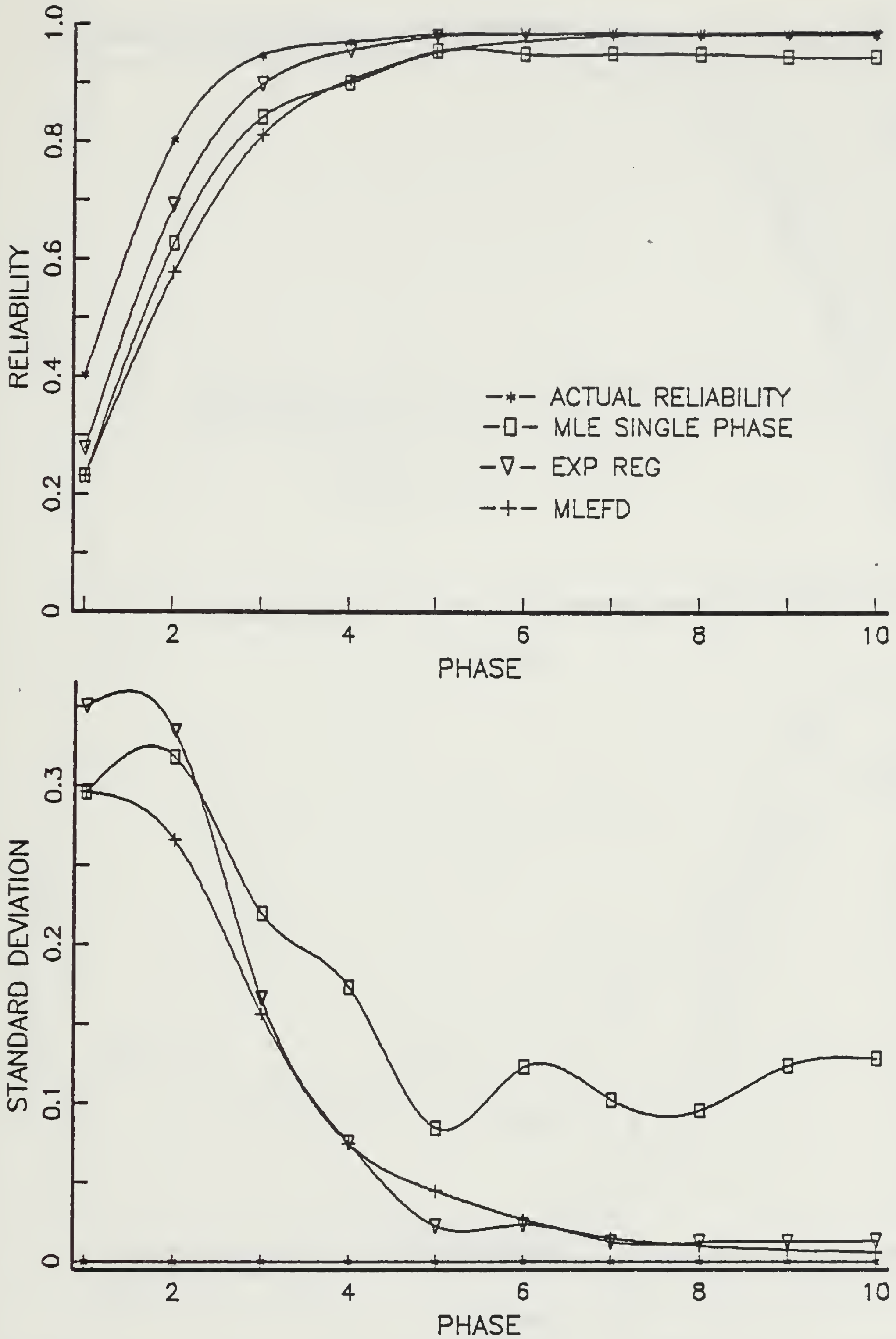

Figure 60. Pattern IV, F = .50, I = 60

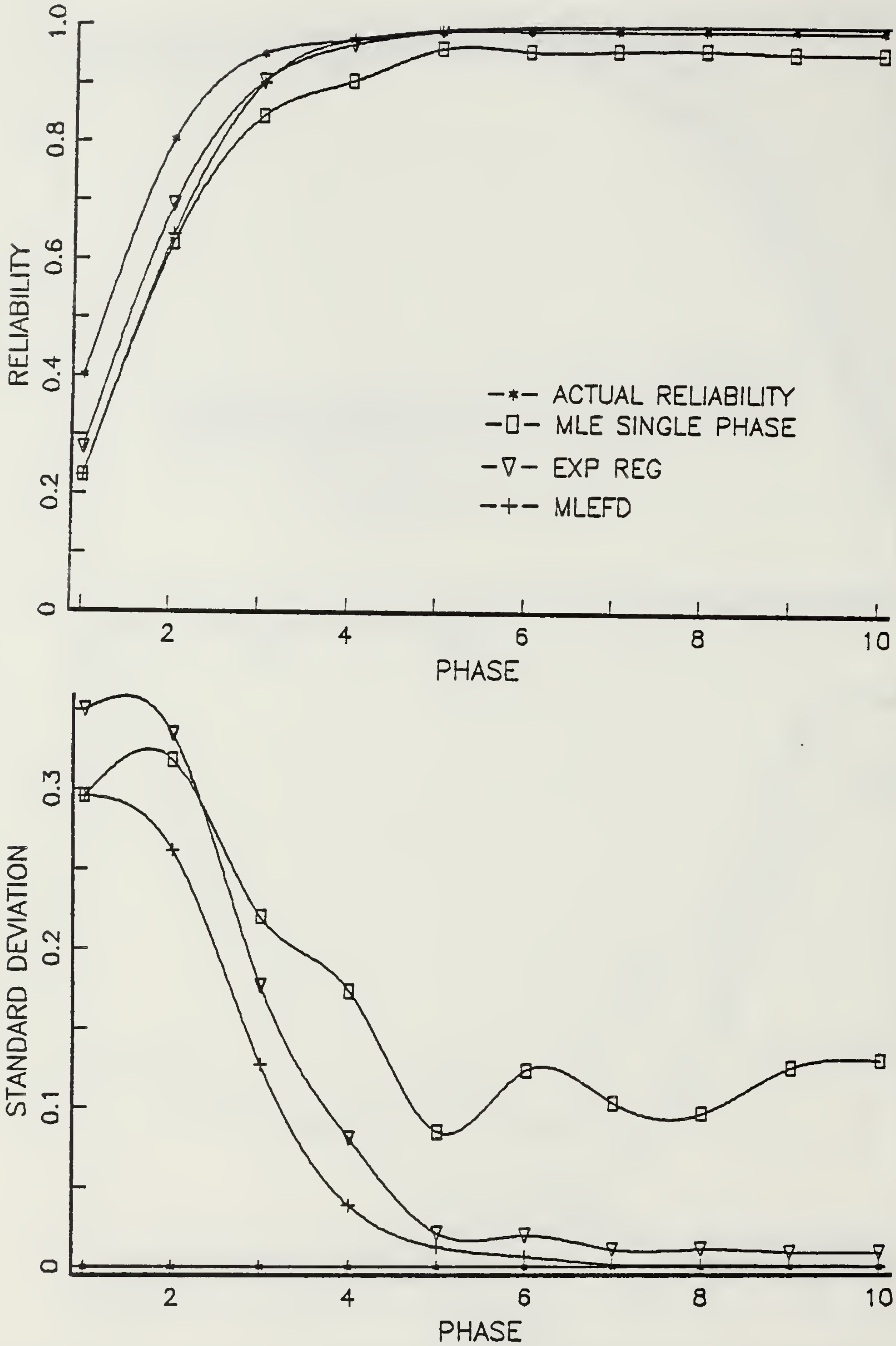

Figure 61. Pattern IV, Lloyd, CI = .8

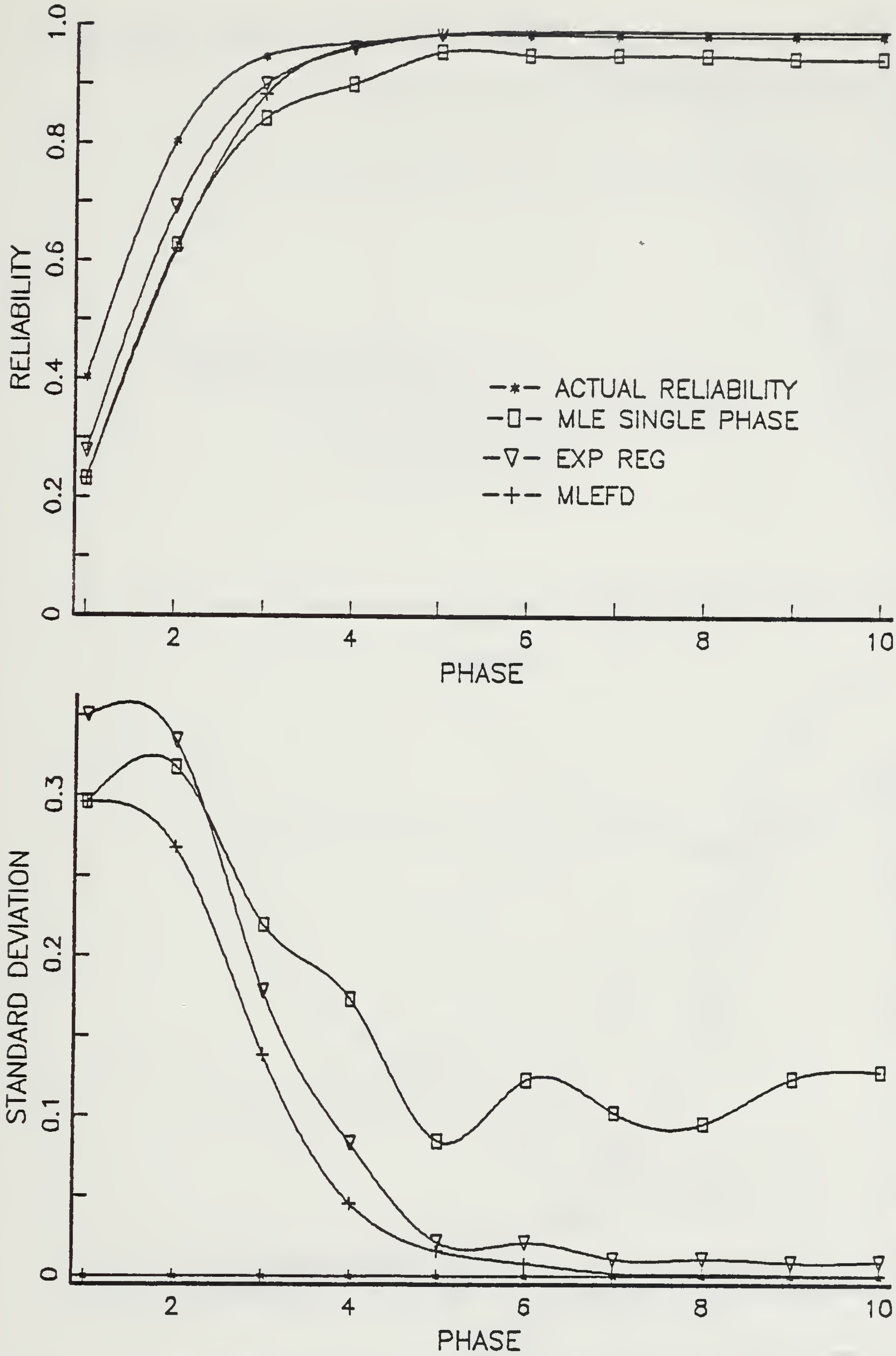

Figure 62. Pattern IV, Lloyd, CI = .9

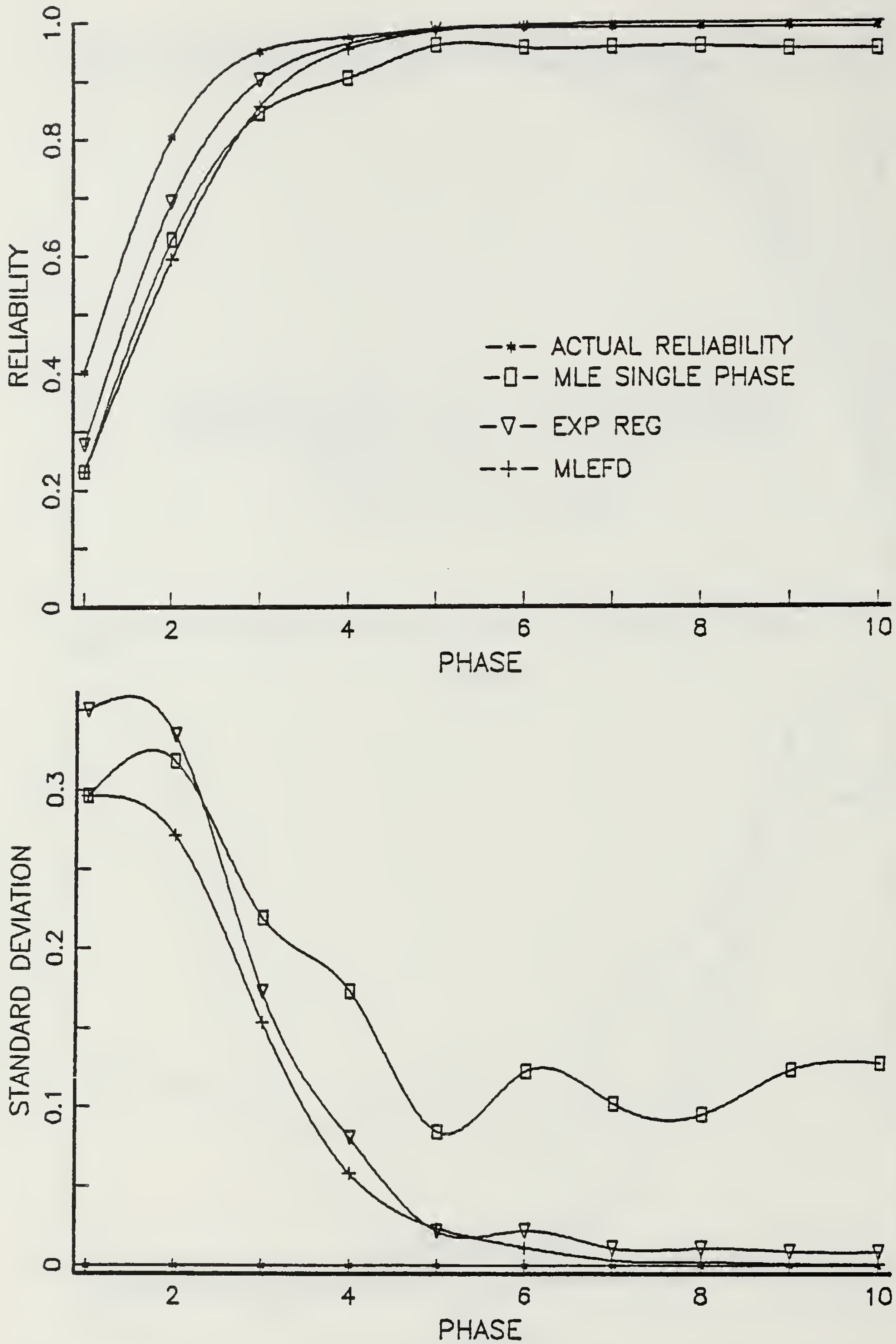

Figure 63. Pattern IV, Lloyd, CI = .99

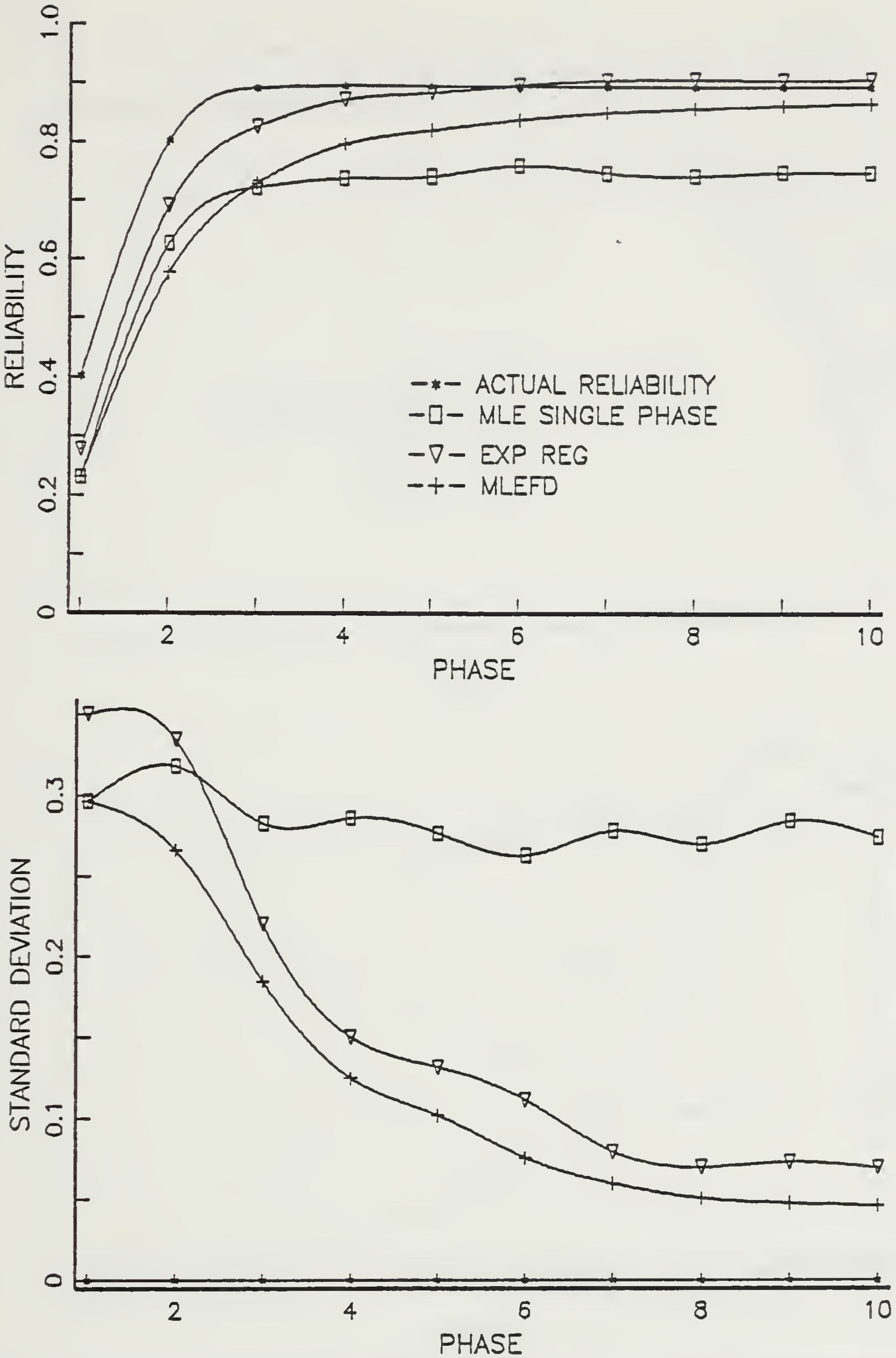

Figure 64. Pattern V, No Discounting

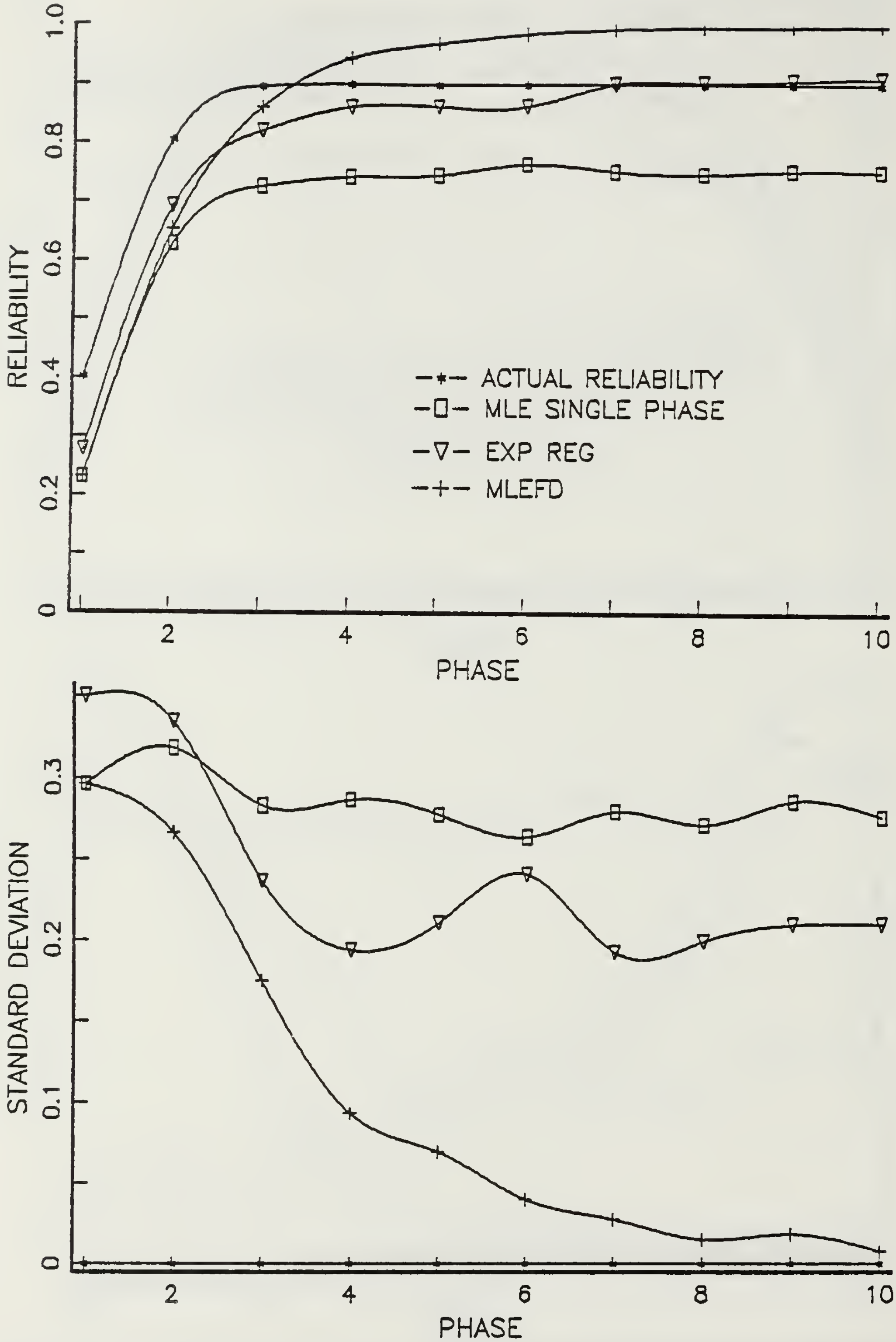

Figure 65. Pattern V, F = .25, I = 1

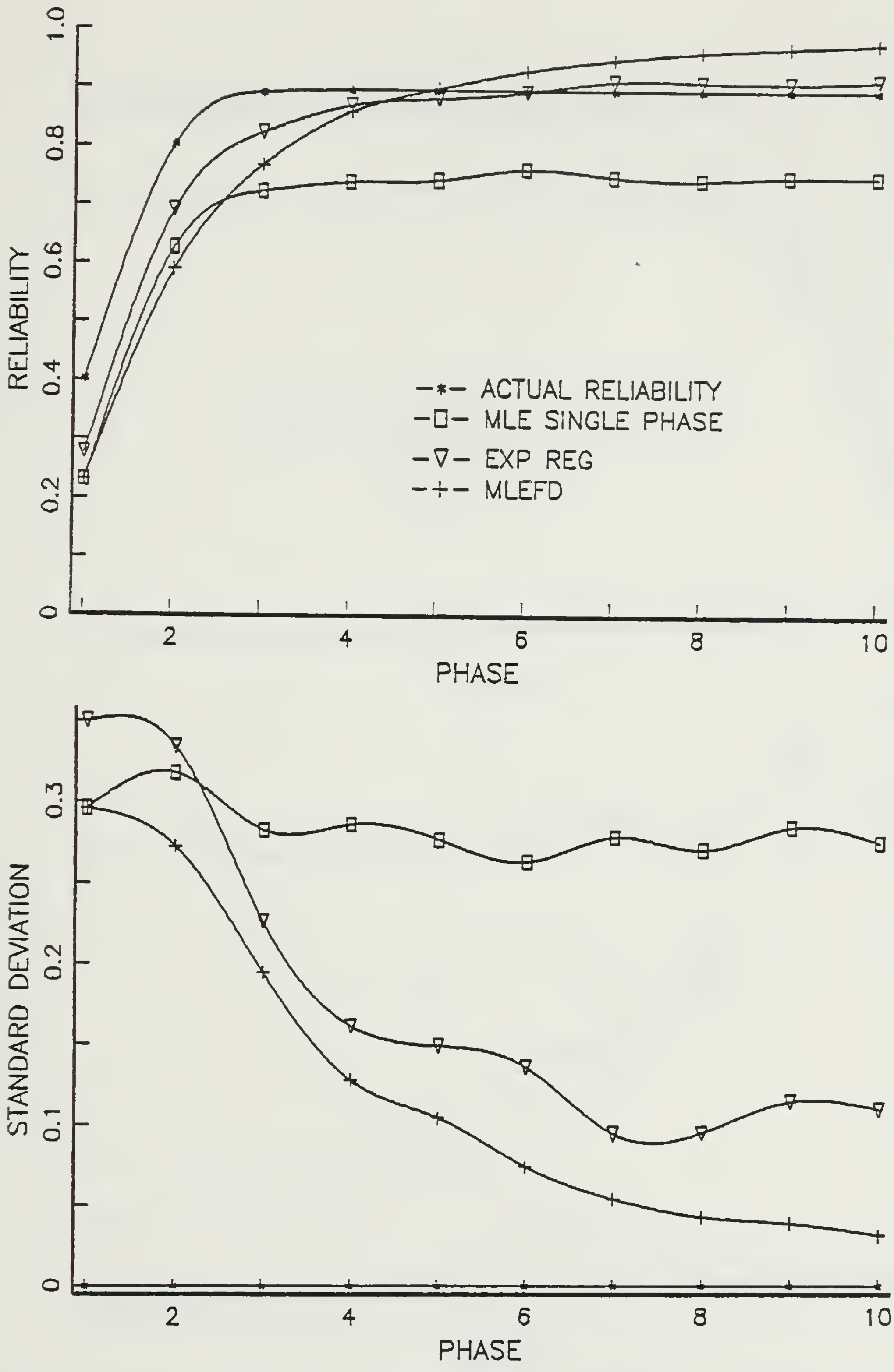

Figure 66. Pattern V, F = .25, I = 3

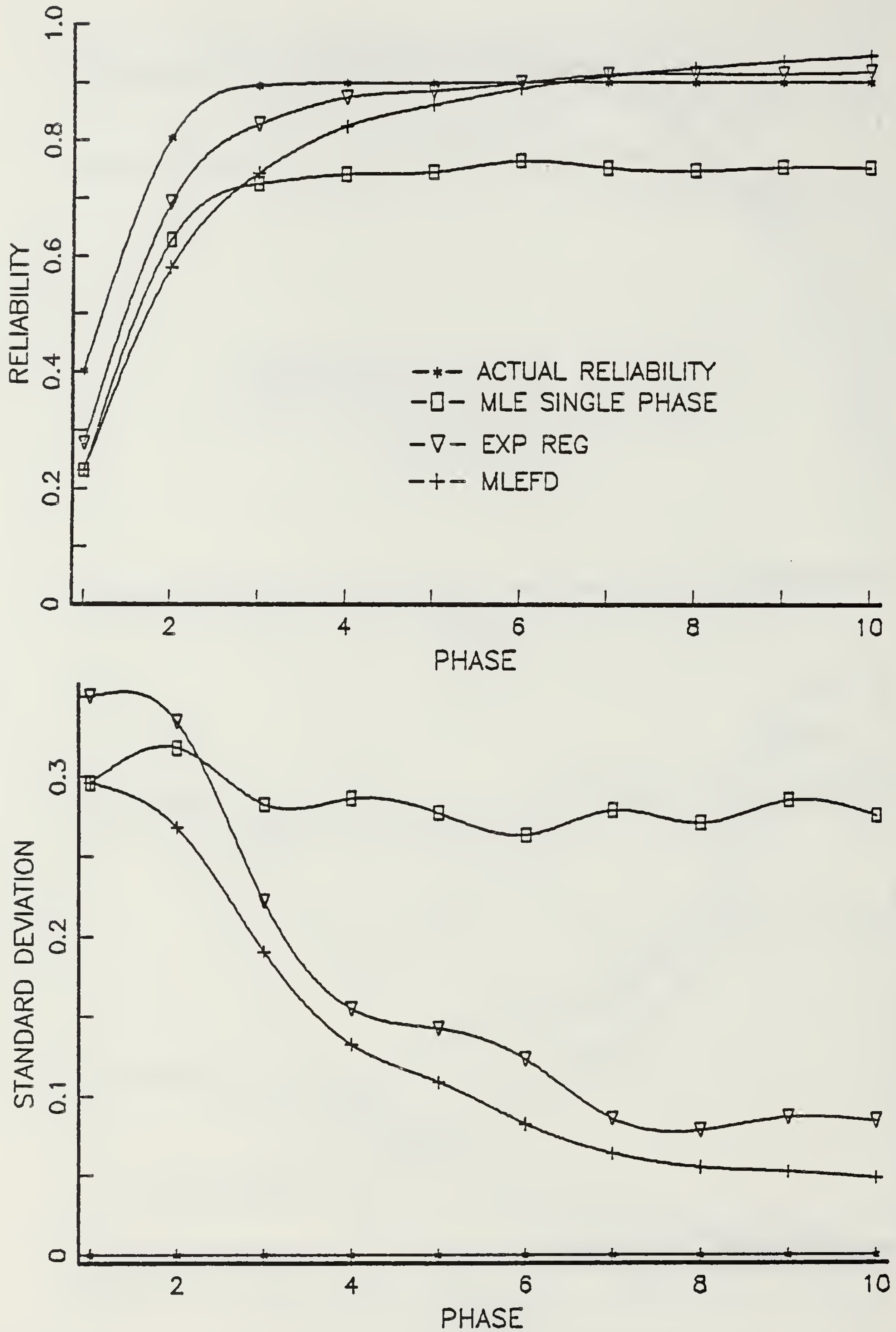

Figure 67. Pattern V, F = .25, I = 6

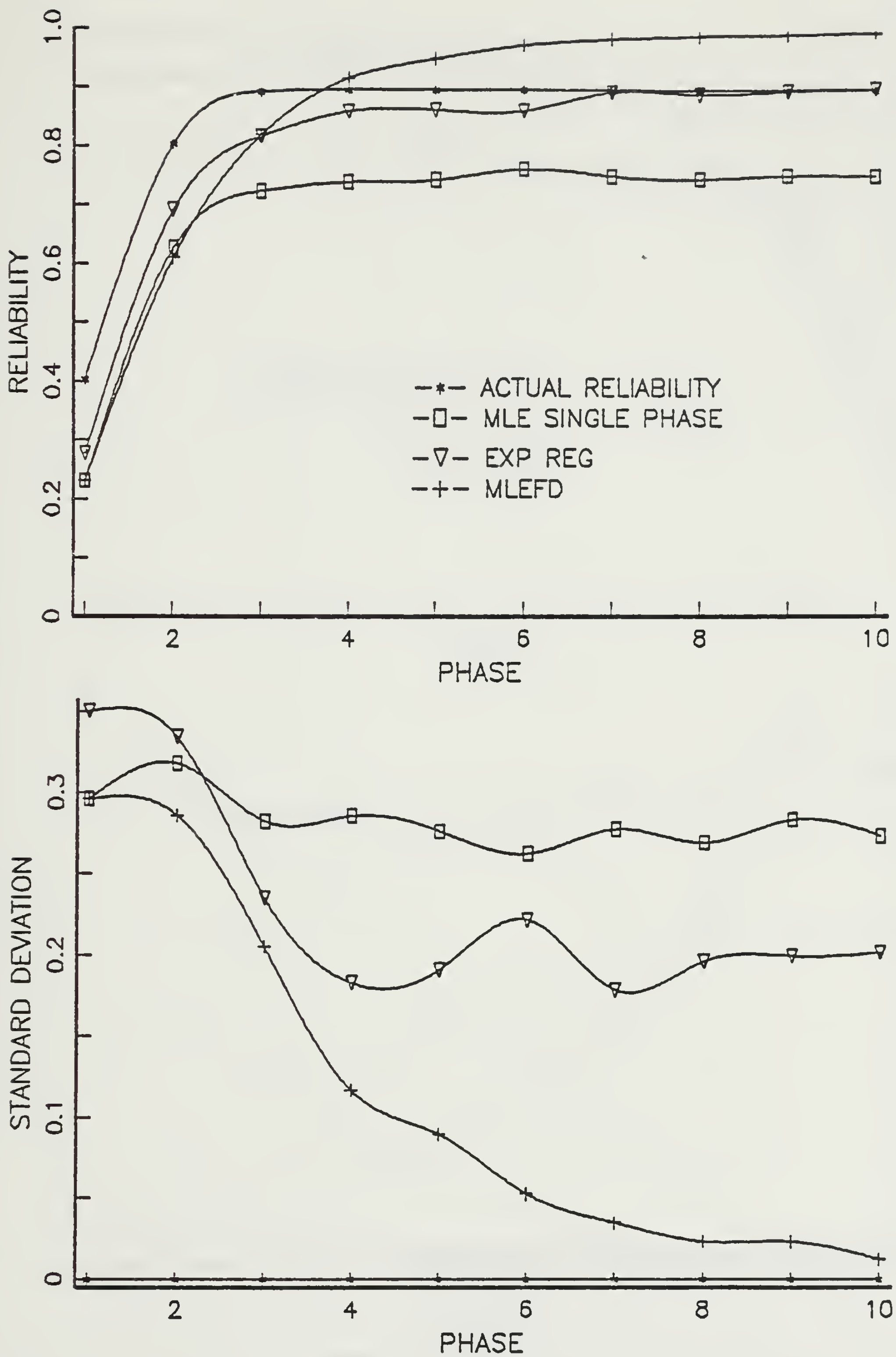

Figure 68. Pattern V, F = .50, I = 3

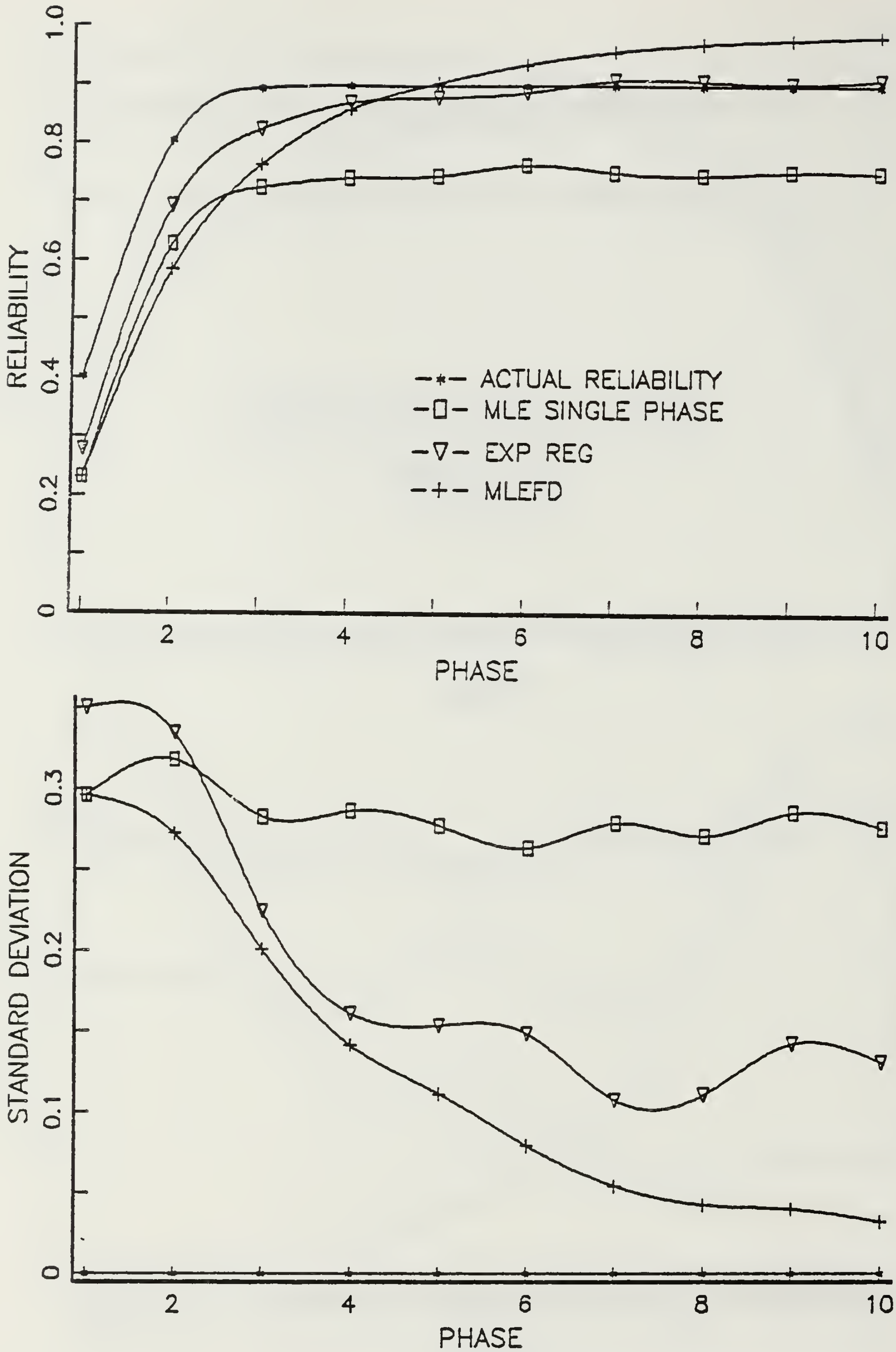

Figure 69. Pattern V, F = .50, I = 6

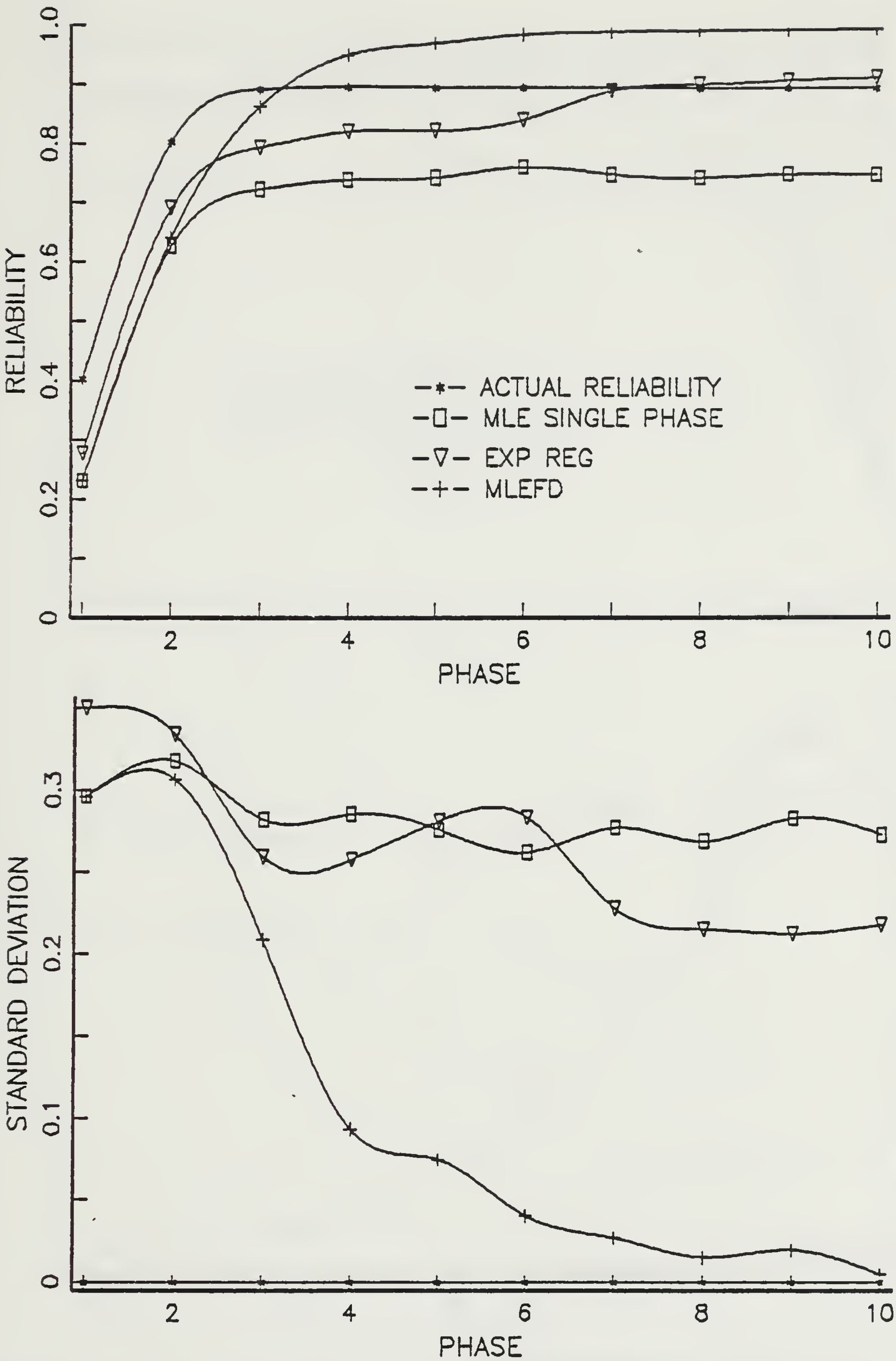

Figure 70. Pattern V, F = .75, I = 3

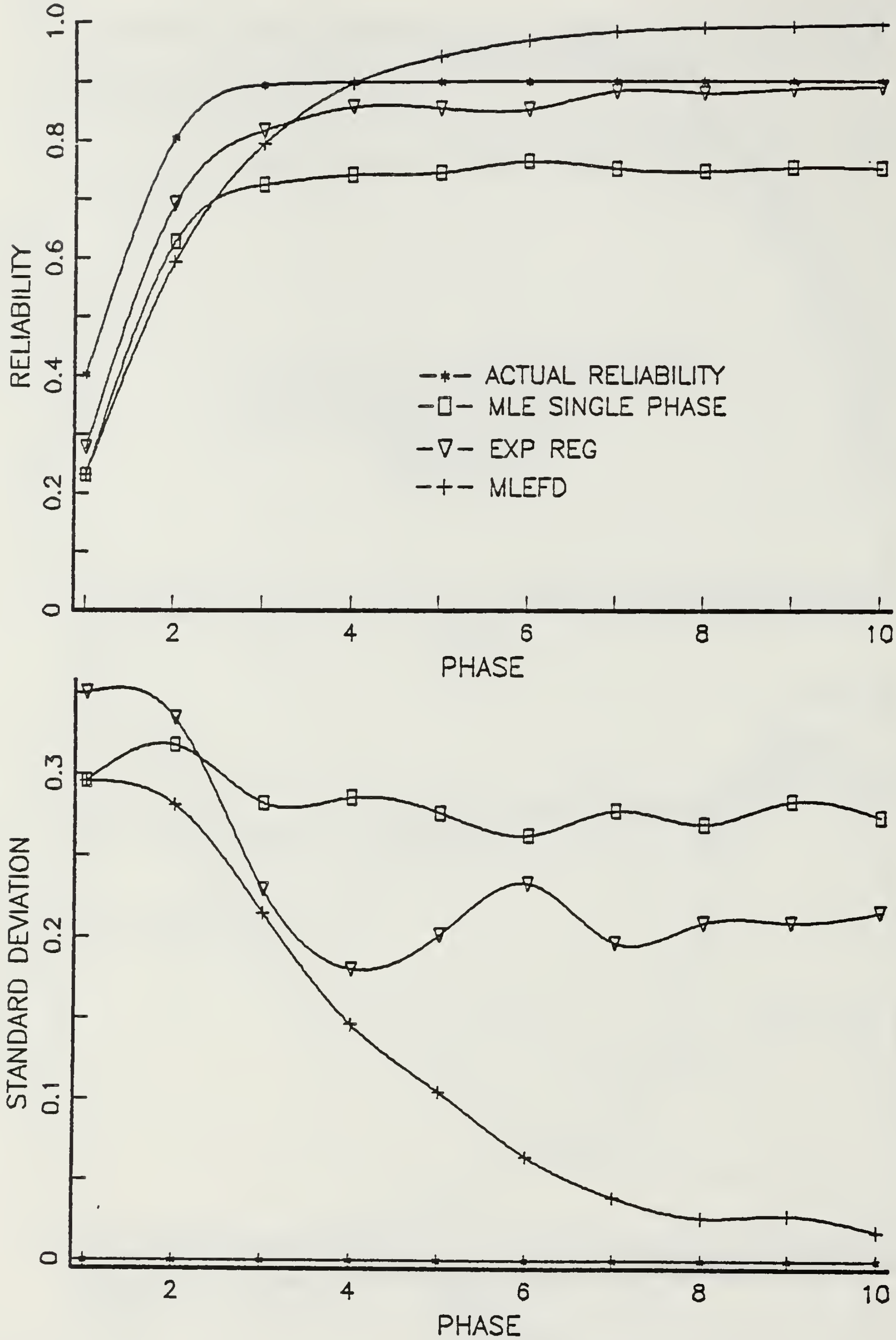

Figure 71. Pattern V, F = .75, I = 6

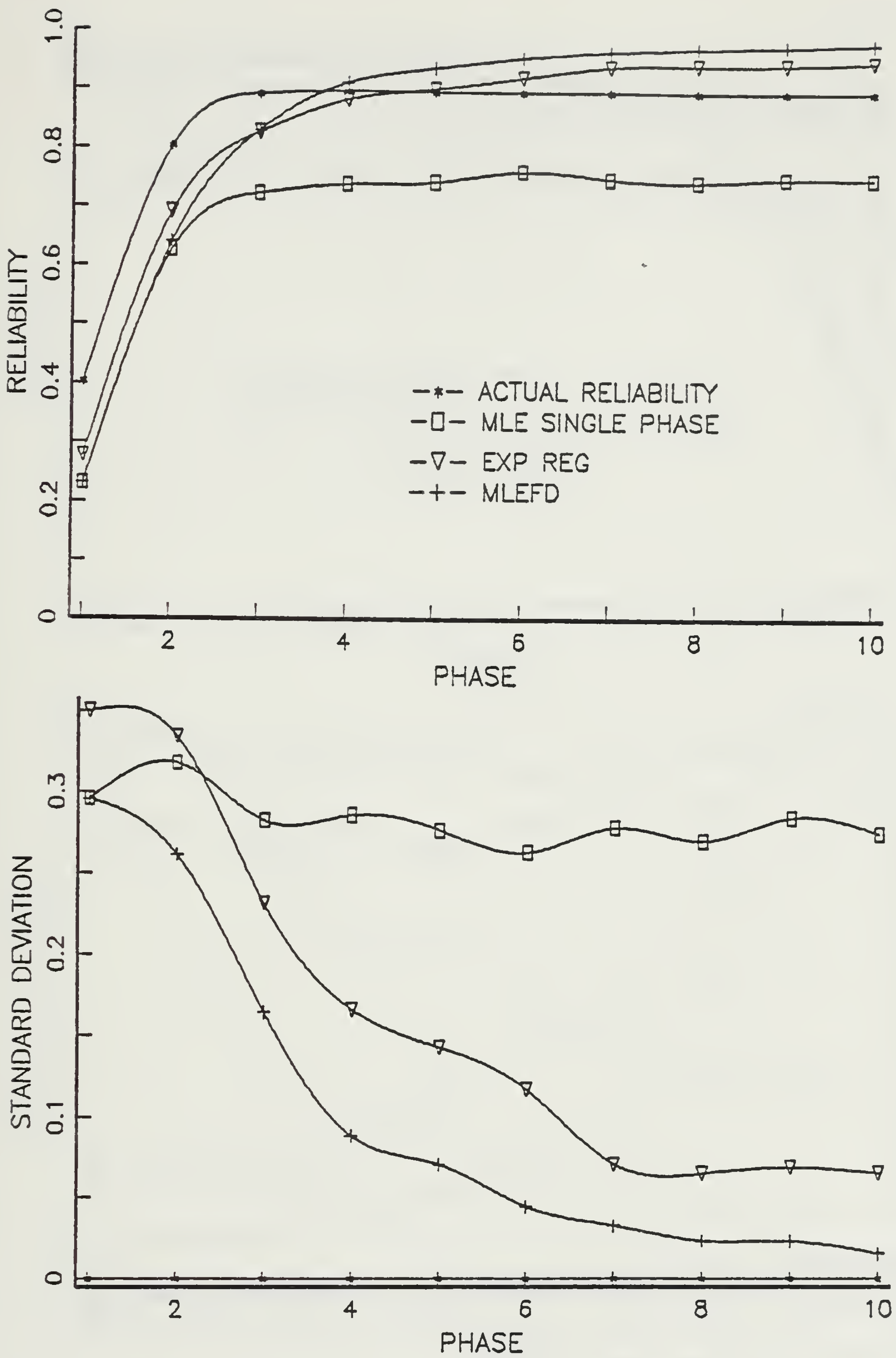

Figure 72. Pattern V, Lloyd, CI = .8

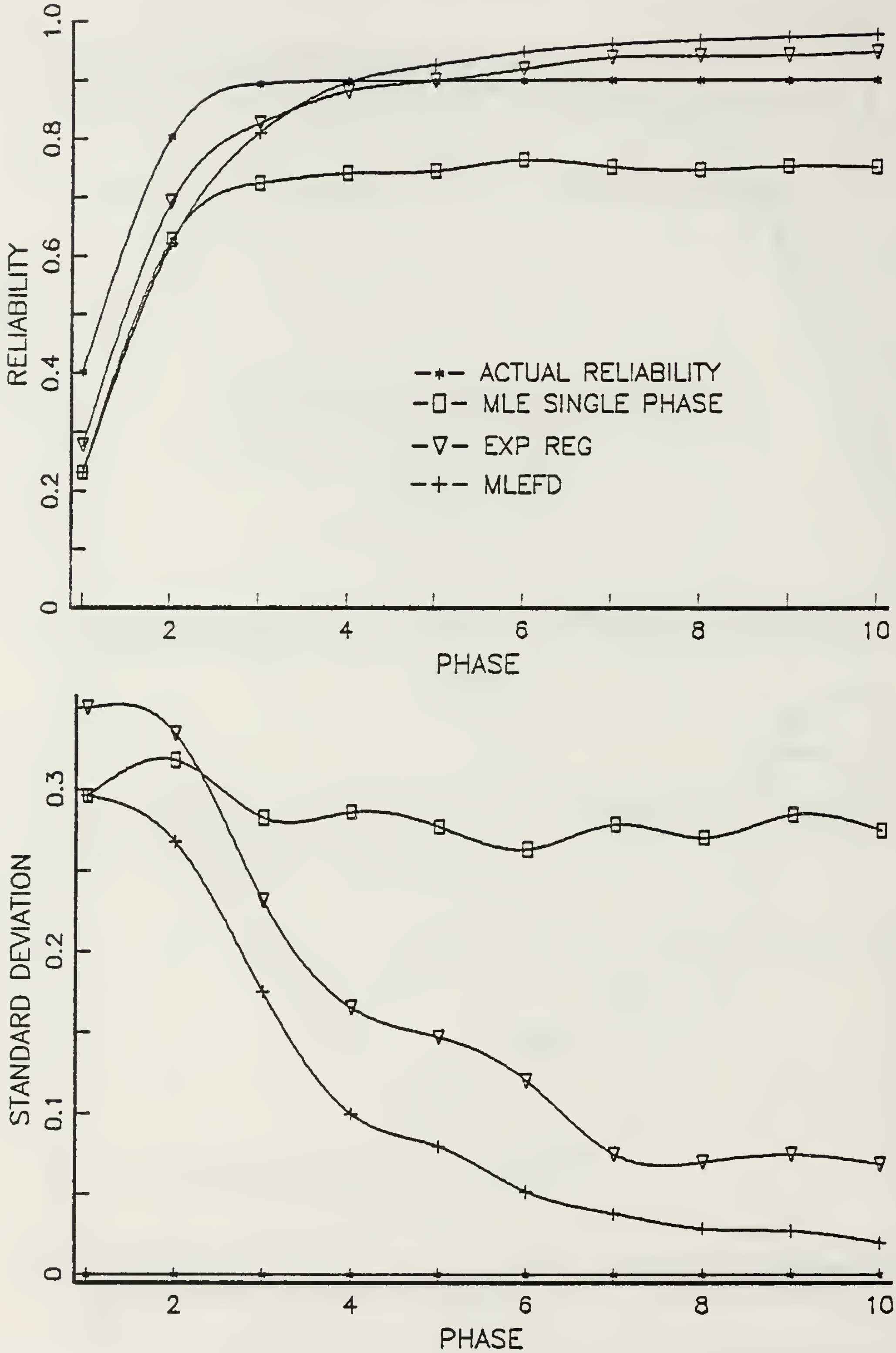

Figure 73. Pattern V, Lloyd, CI = .9

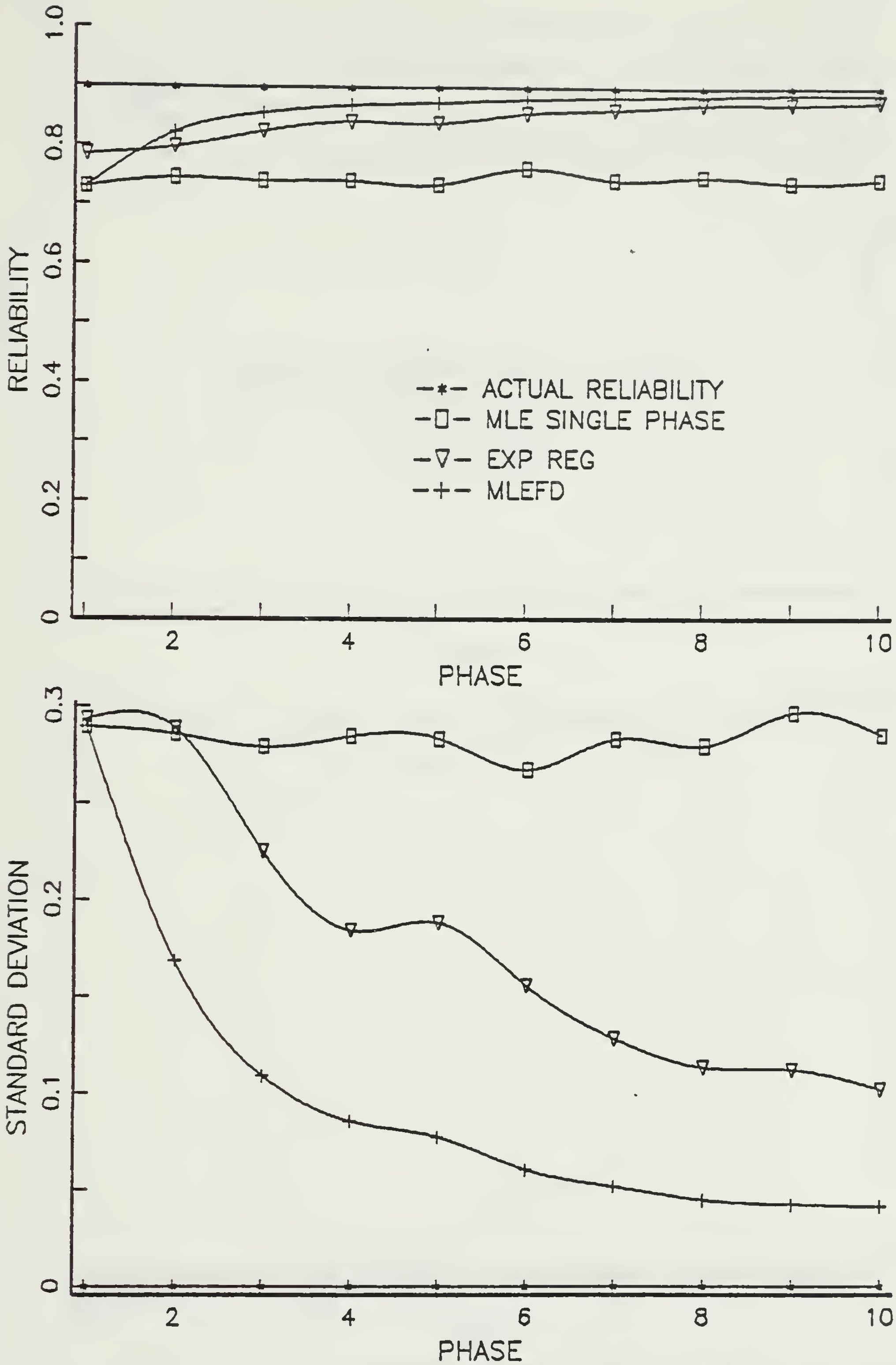

Figure 74. Pattern VI, No Discounting

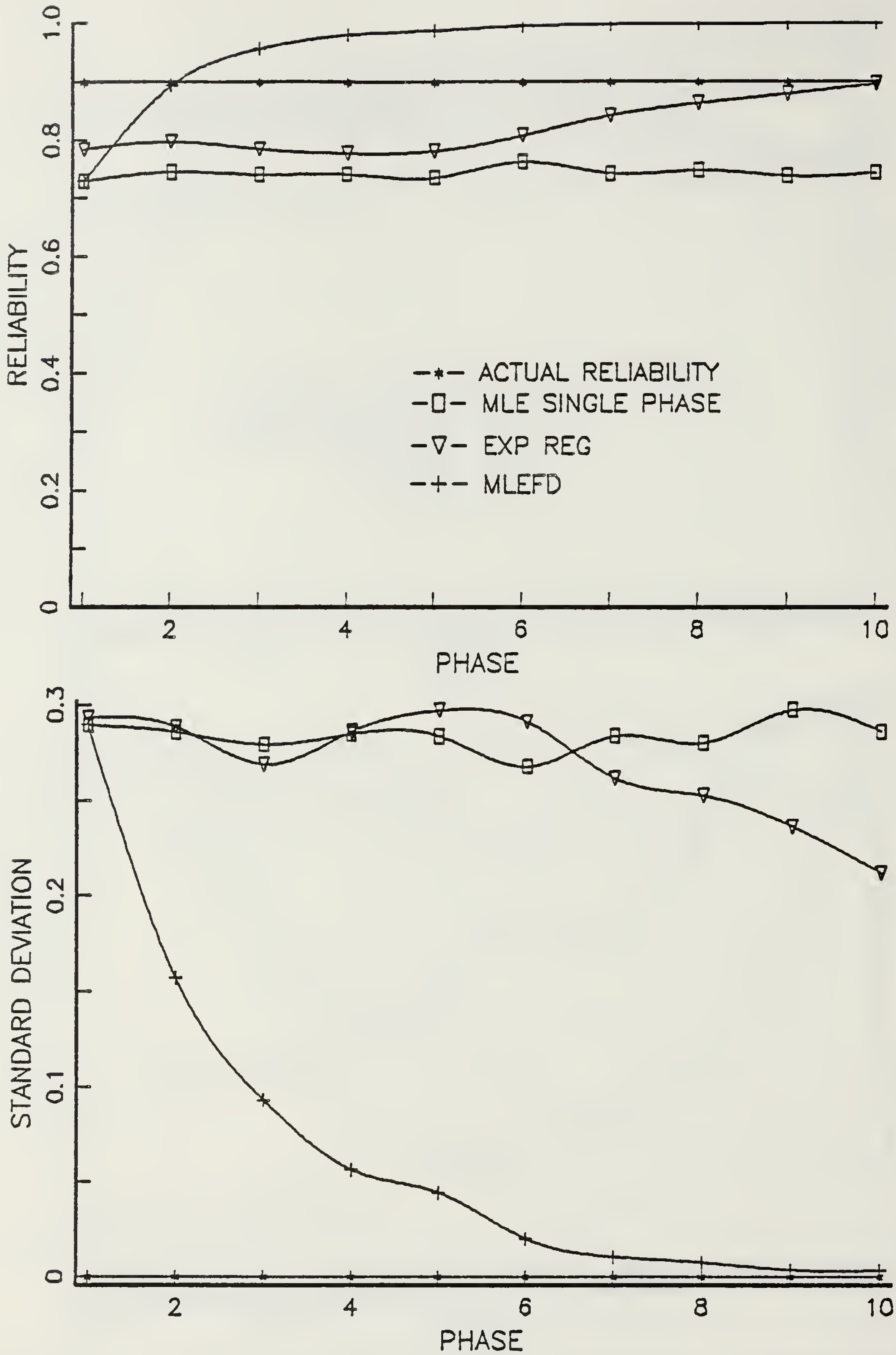

Figure 75. Pattern VI, F = .25, I = 1

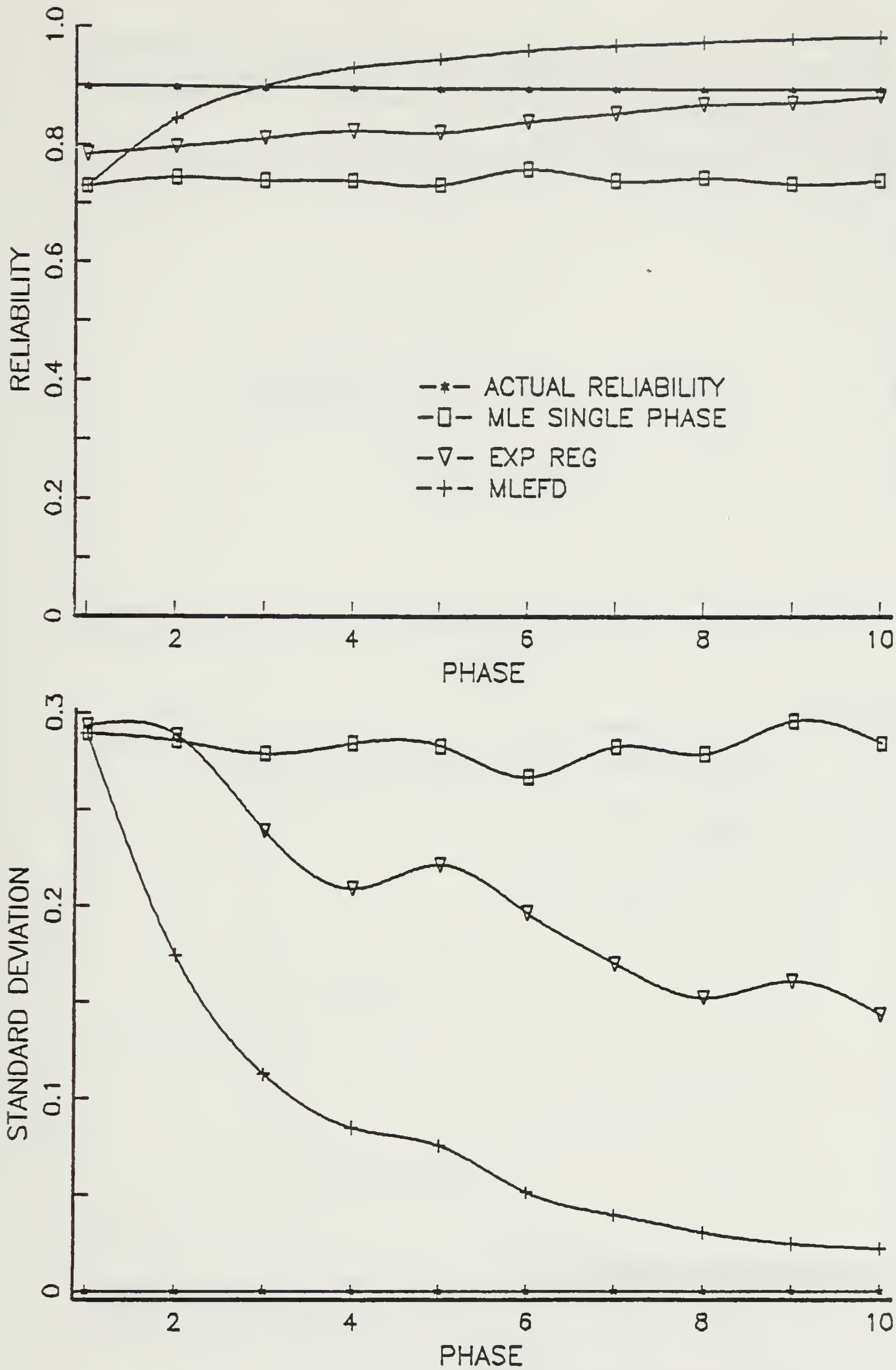

Figure 76. Pattern VI, F = .25, I = 3

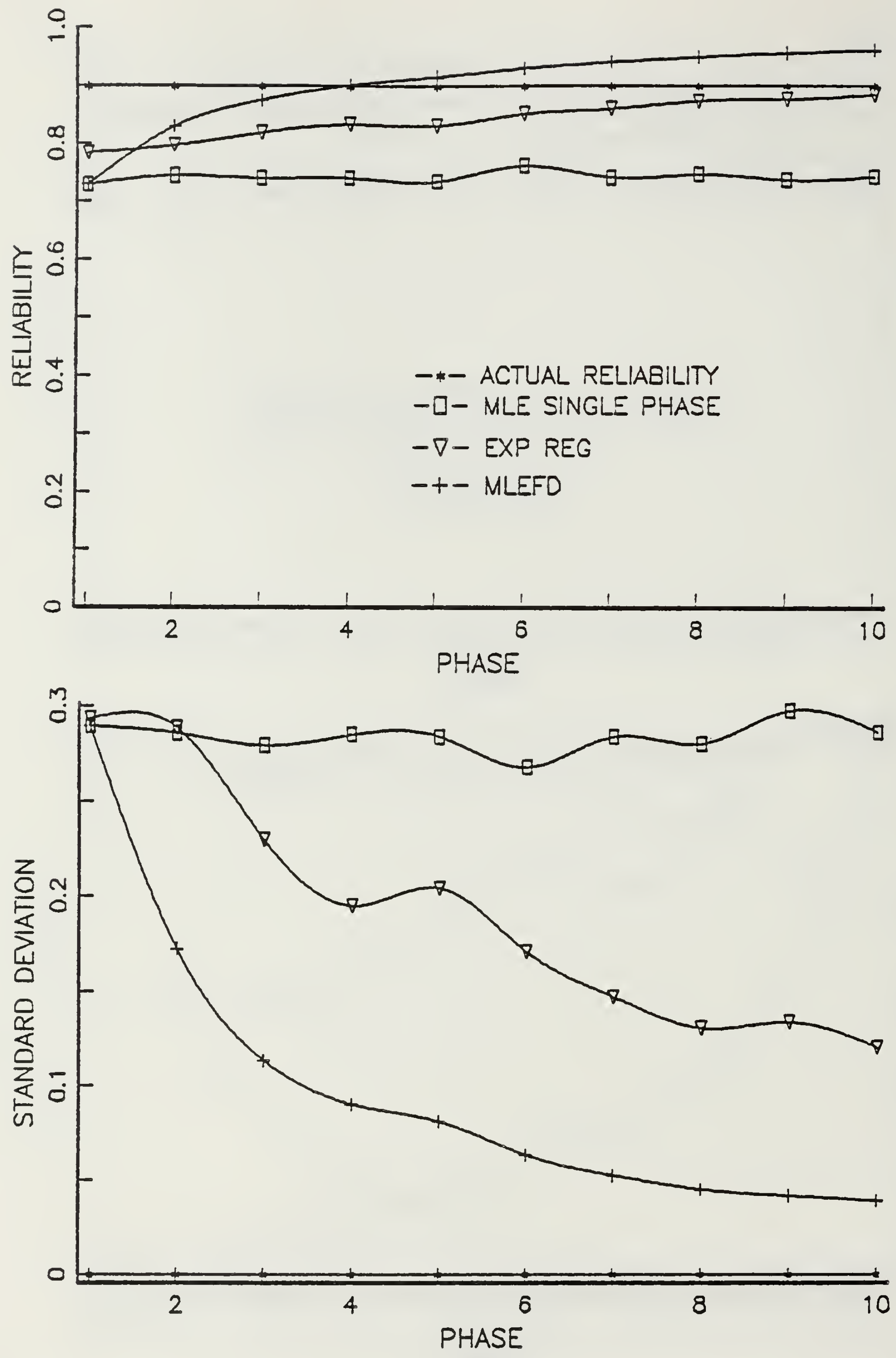

Figure 77. Pattern VI, F = .25, I = 6

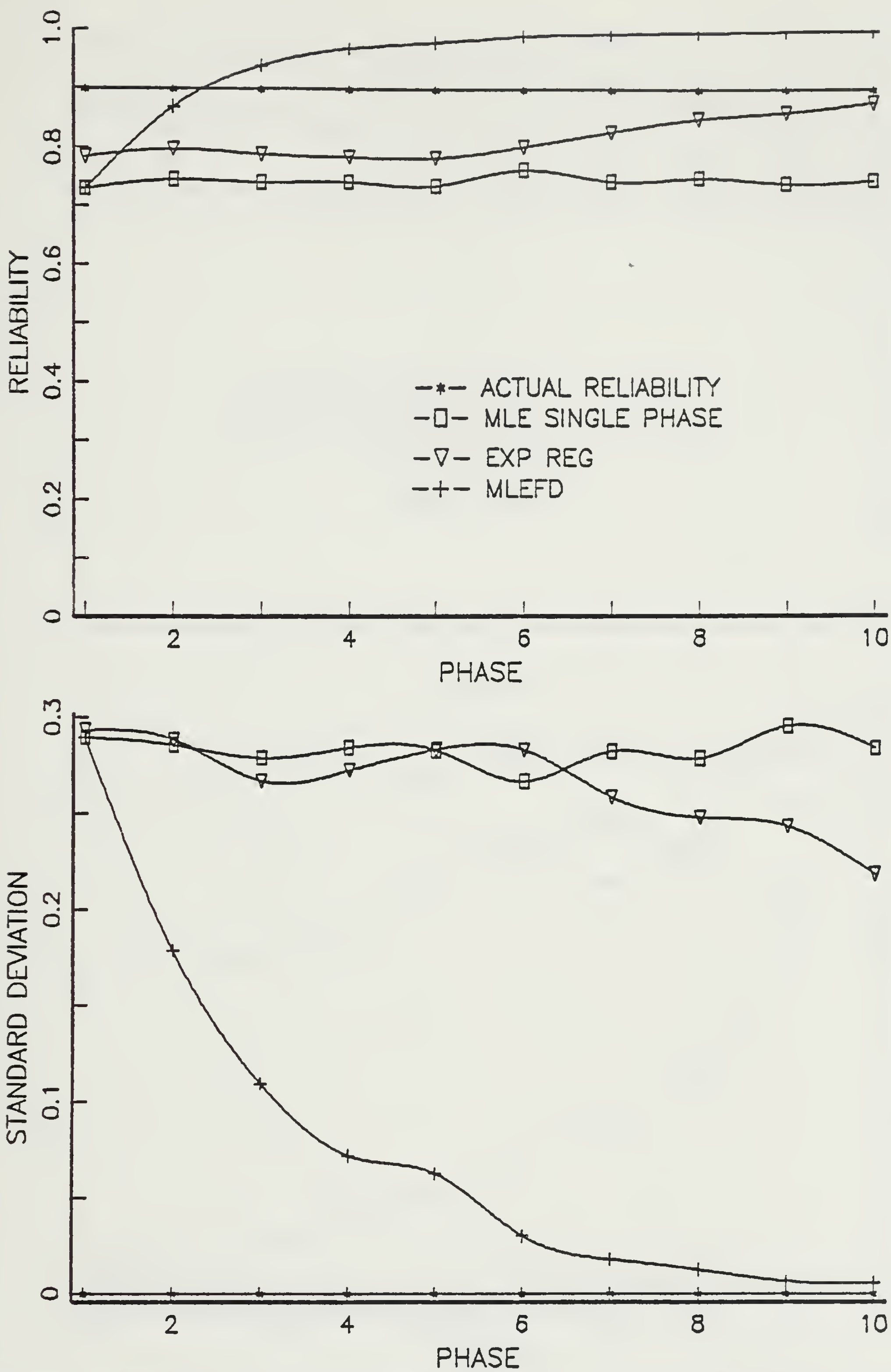

Figure 78. Pattern VI, F = .50, I = 3

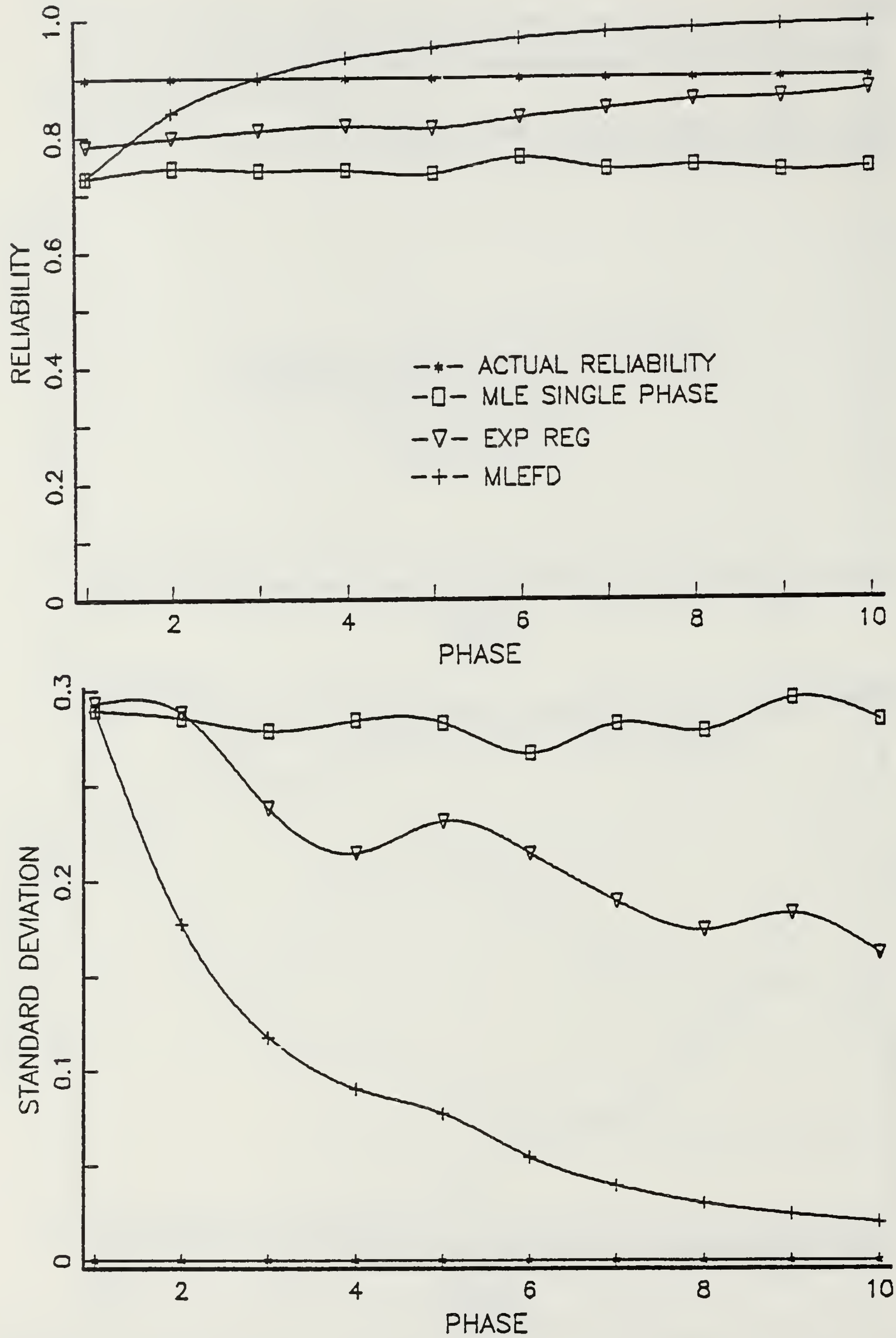

Figure 79. Pattern VI, F = .50, I = 6

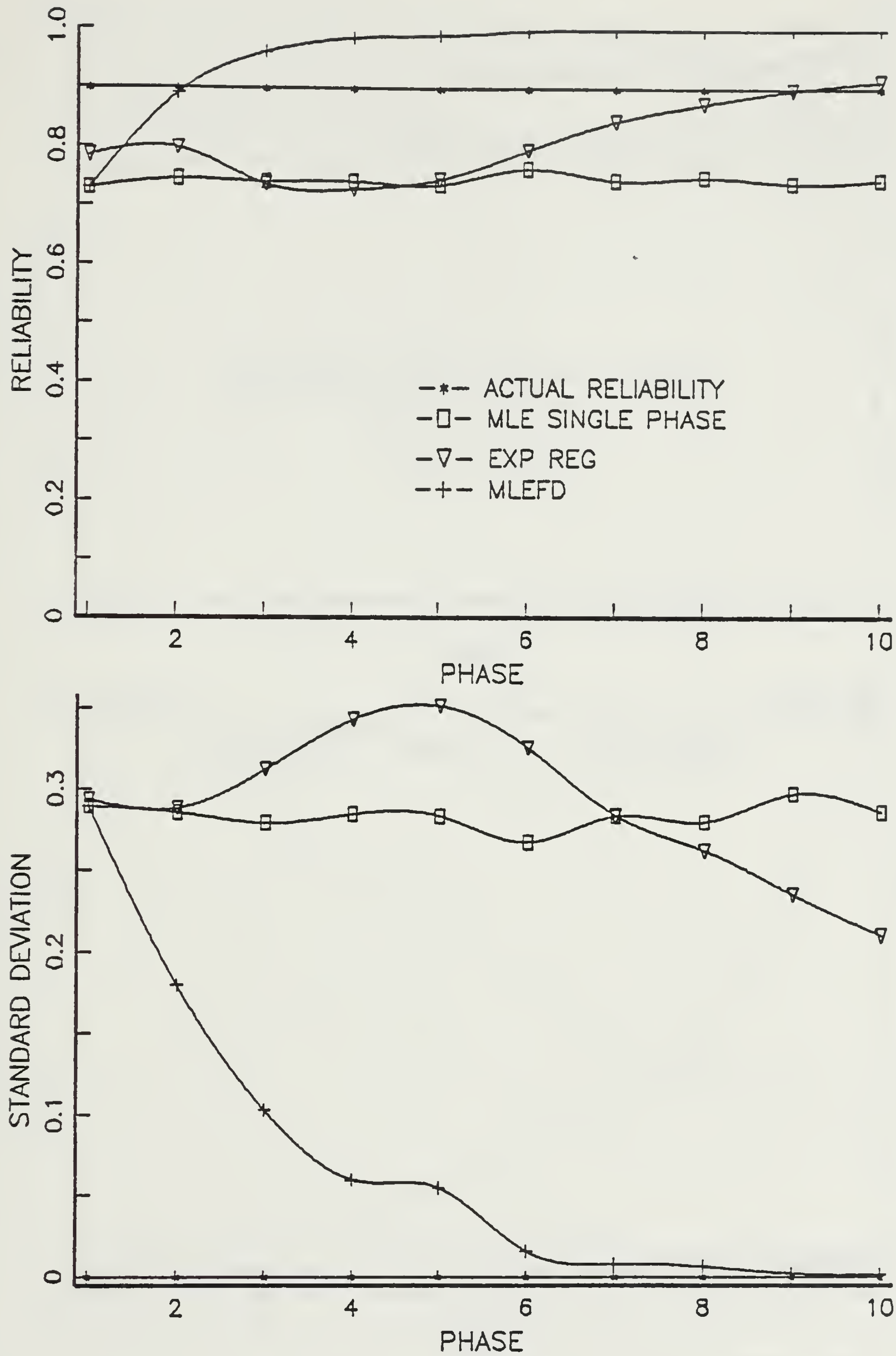

Figure 80. Pattern VI, F = .75, I = 3

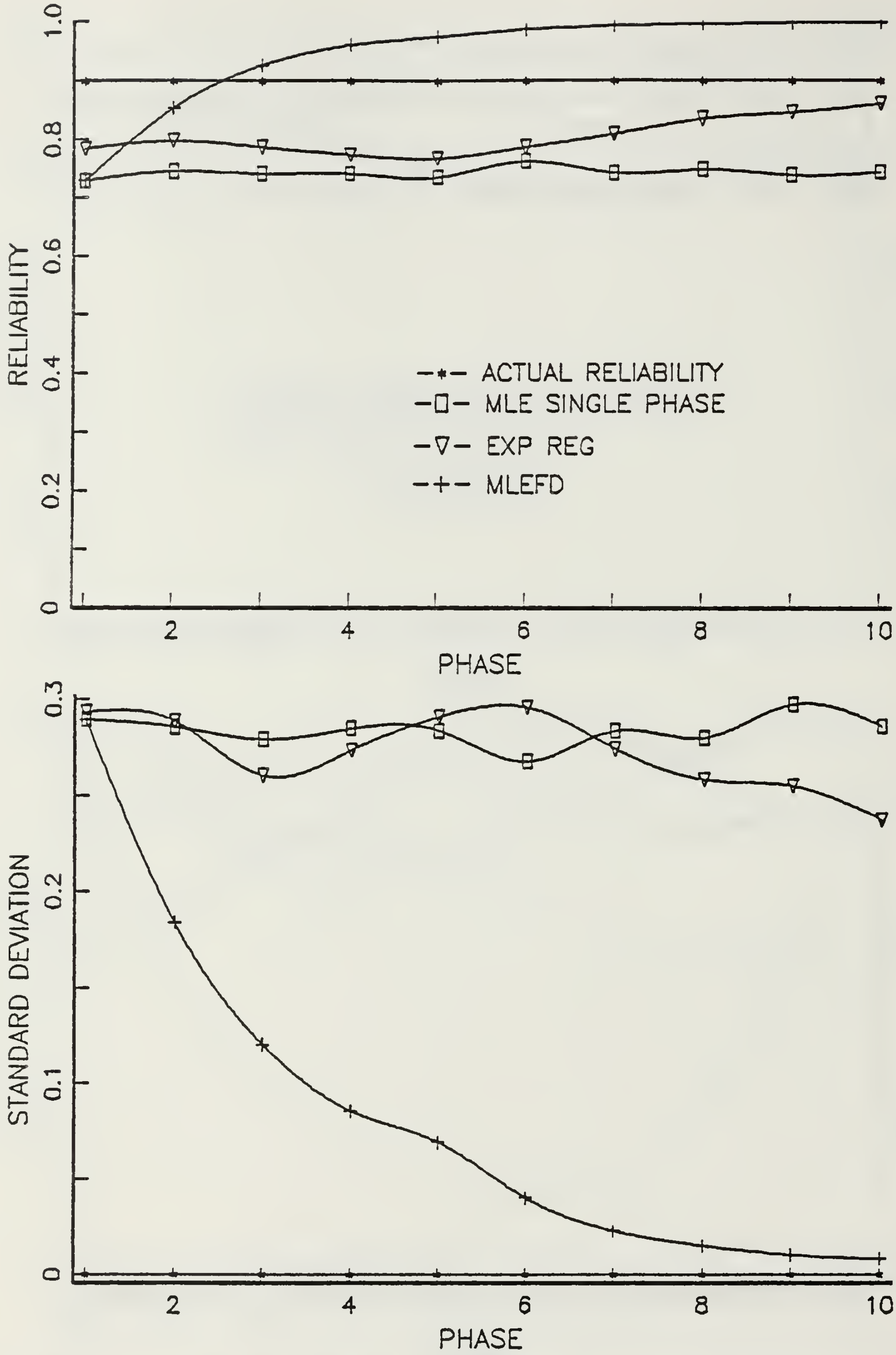

Figure 81. Pattern VI, F = .75, I = 6

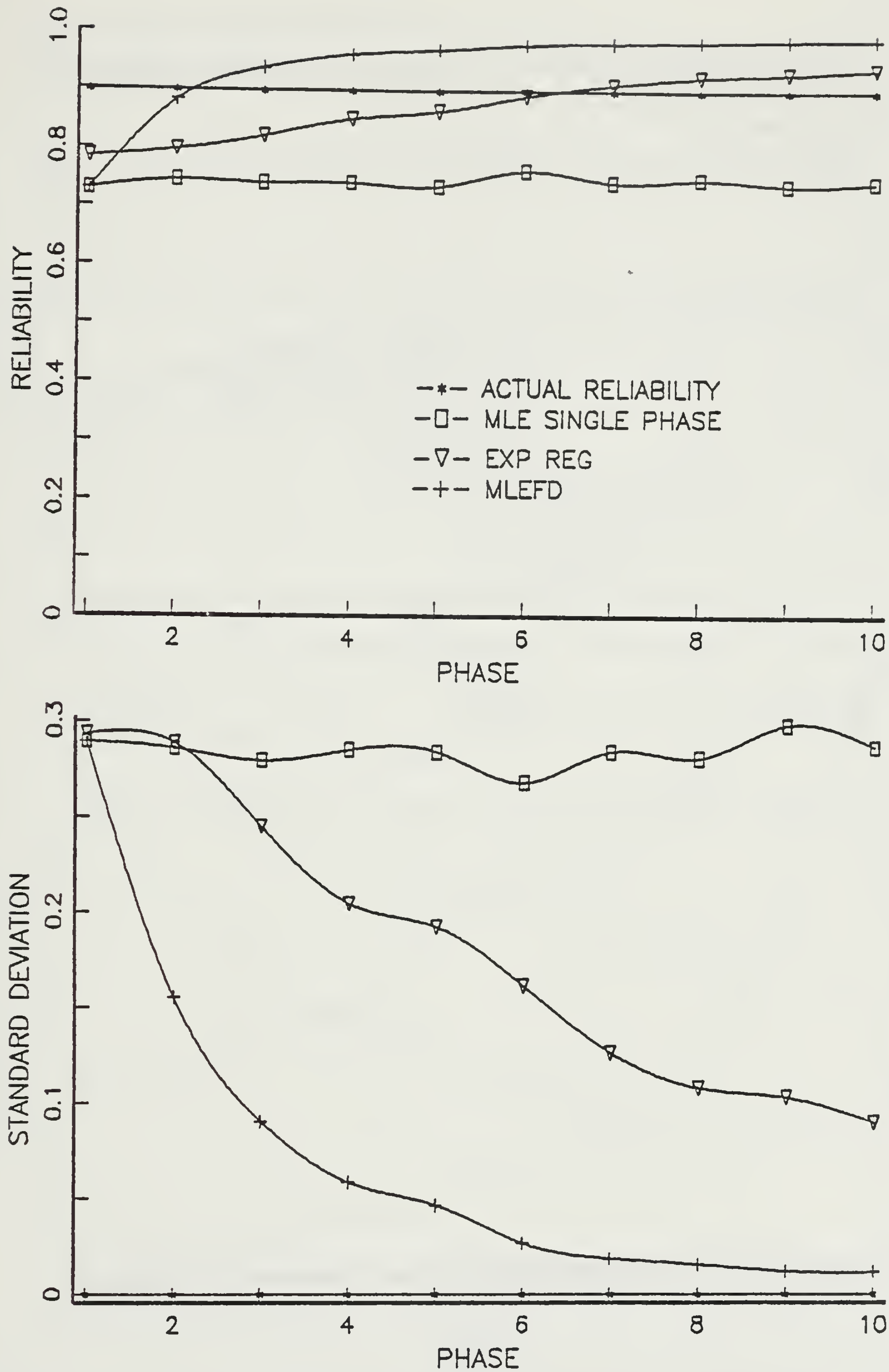

Figure 82. Pattern VI, Lloyd, CI = .8

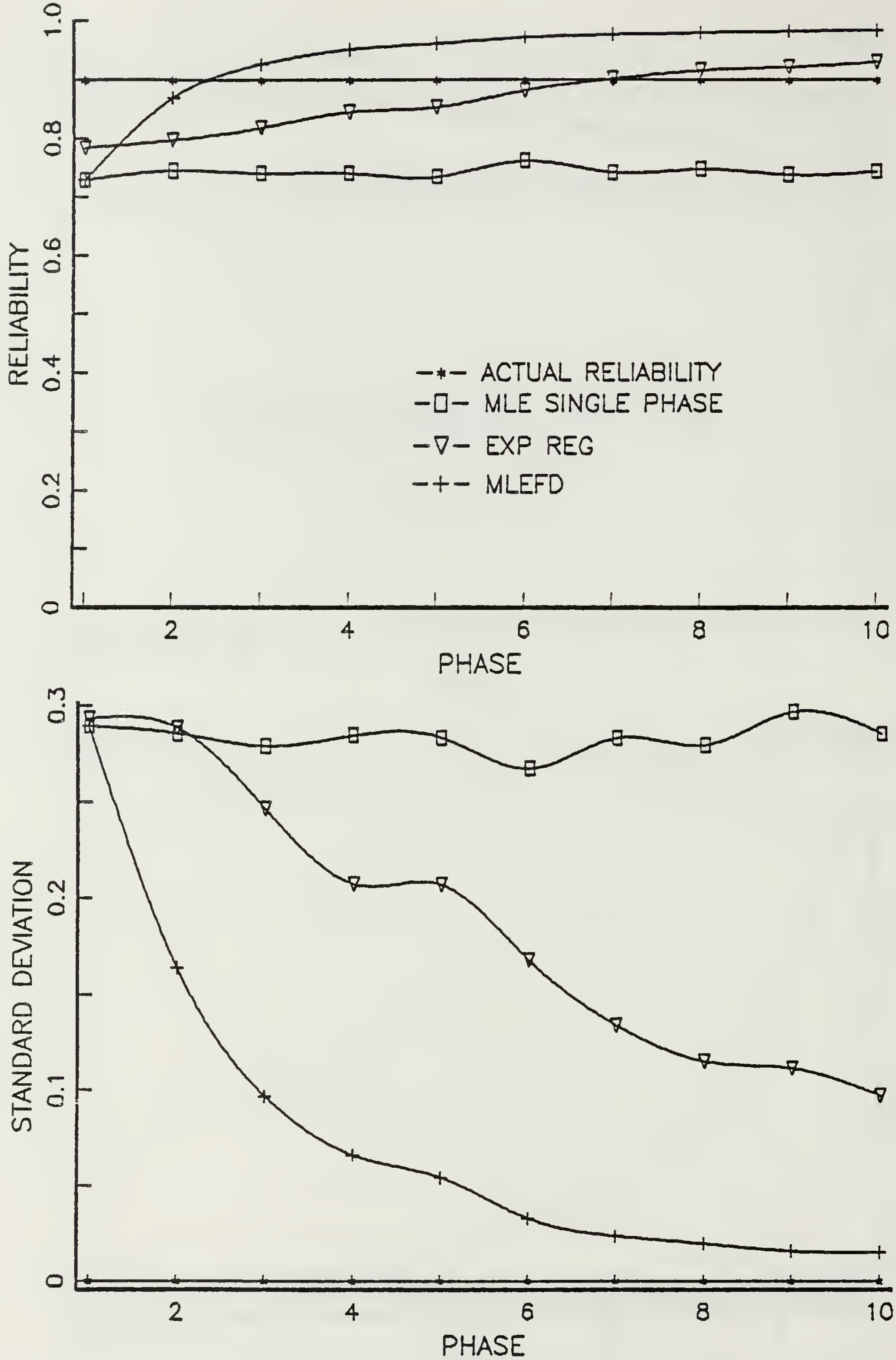

Figure 83. Pattern VI, Lloyd, CI = .9

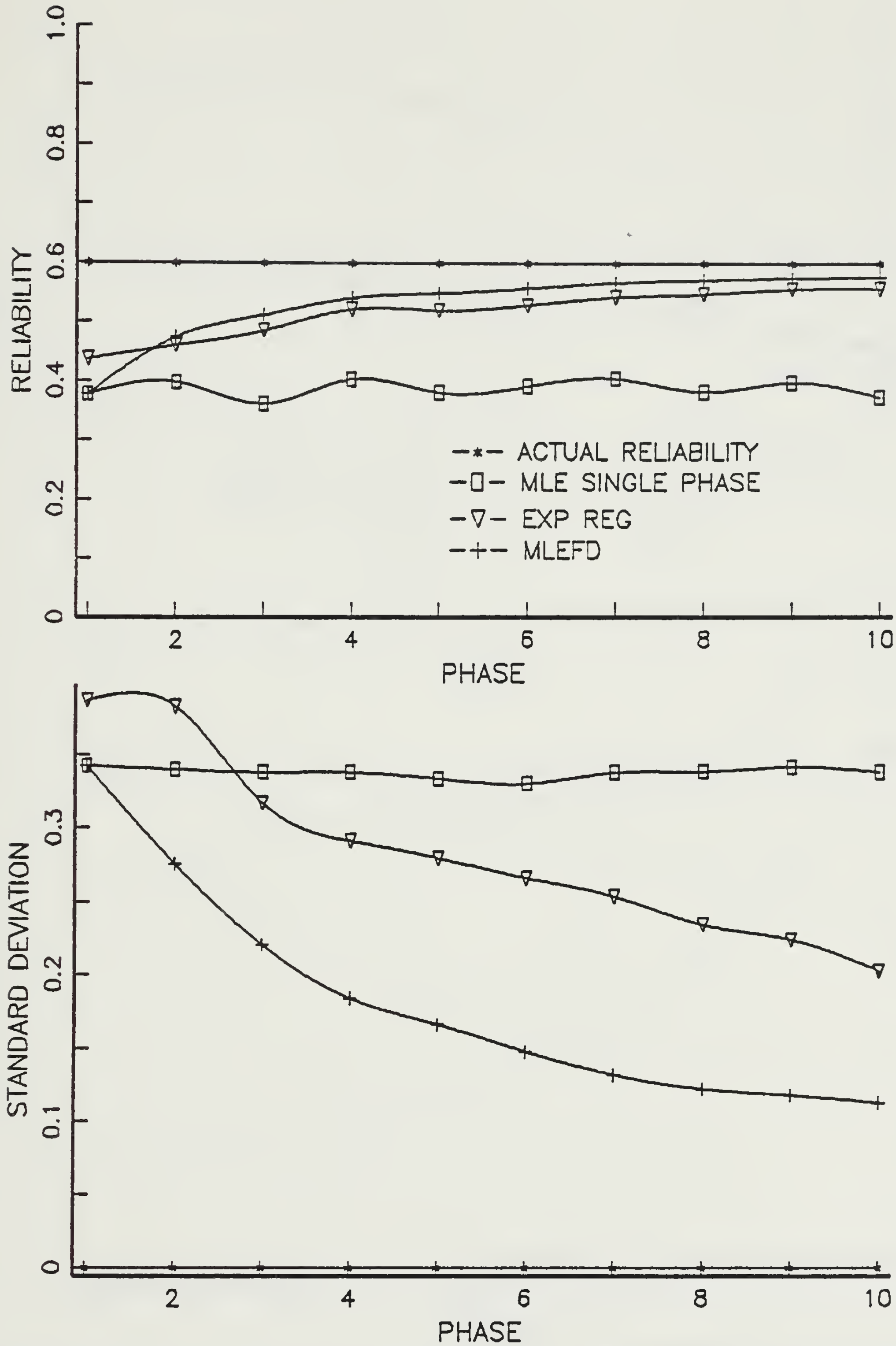

Figure 84. Pattern VII, No Discounting

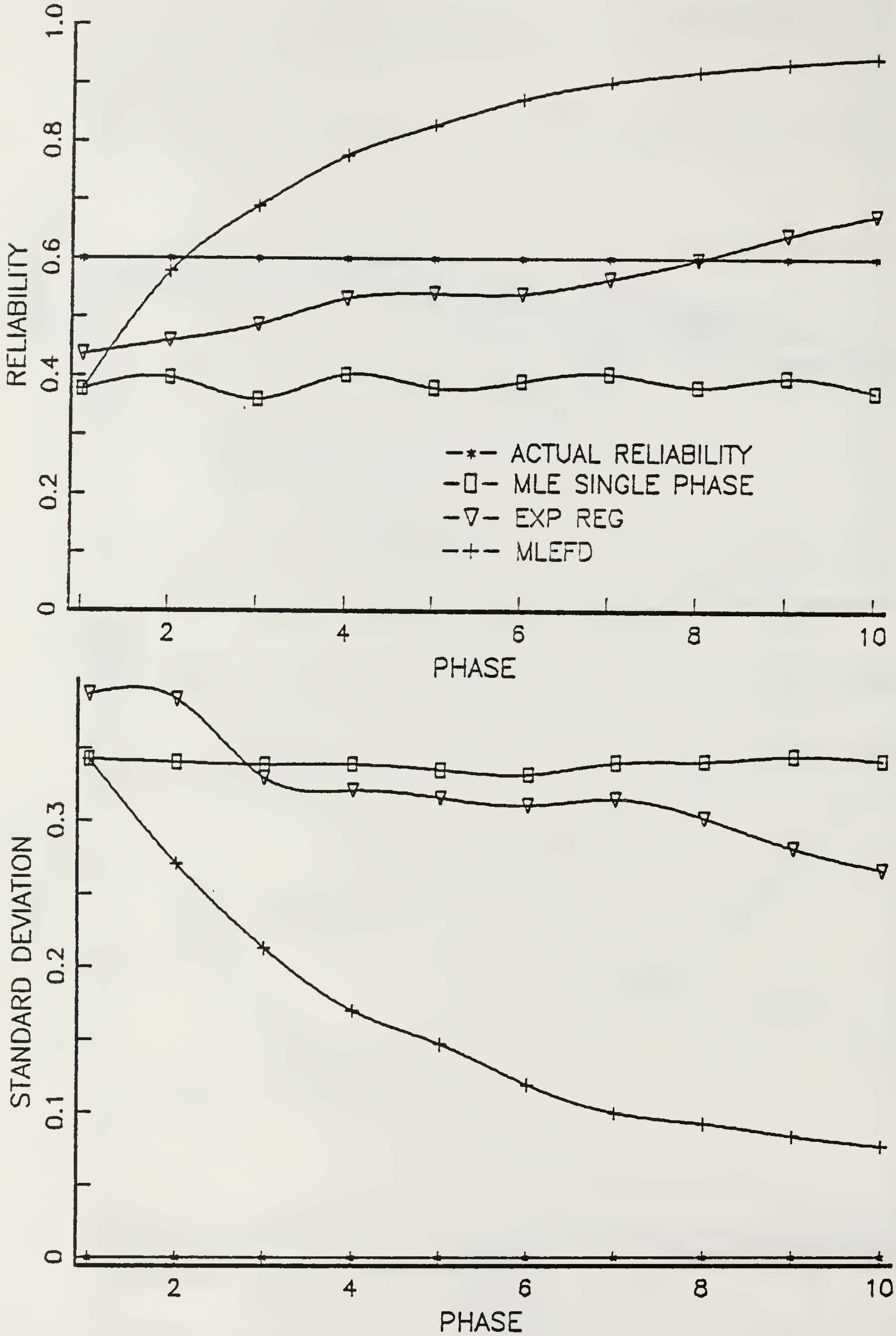

Figure 85. Pattern VII, F = .25, I = 1

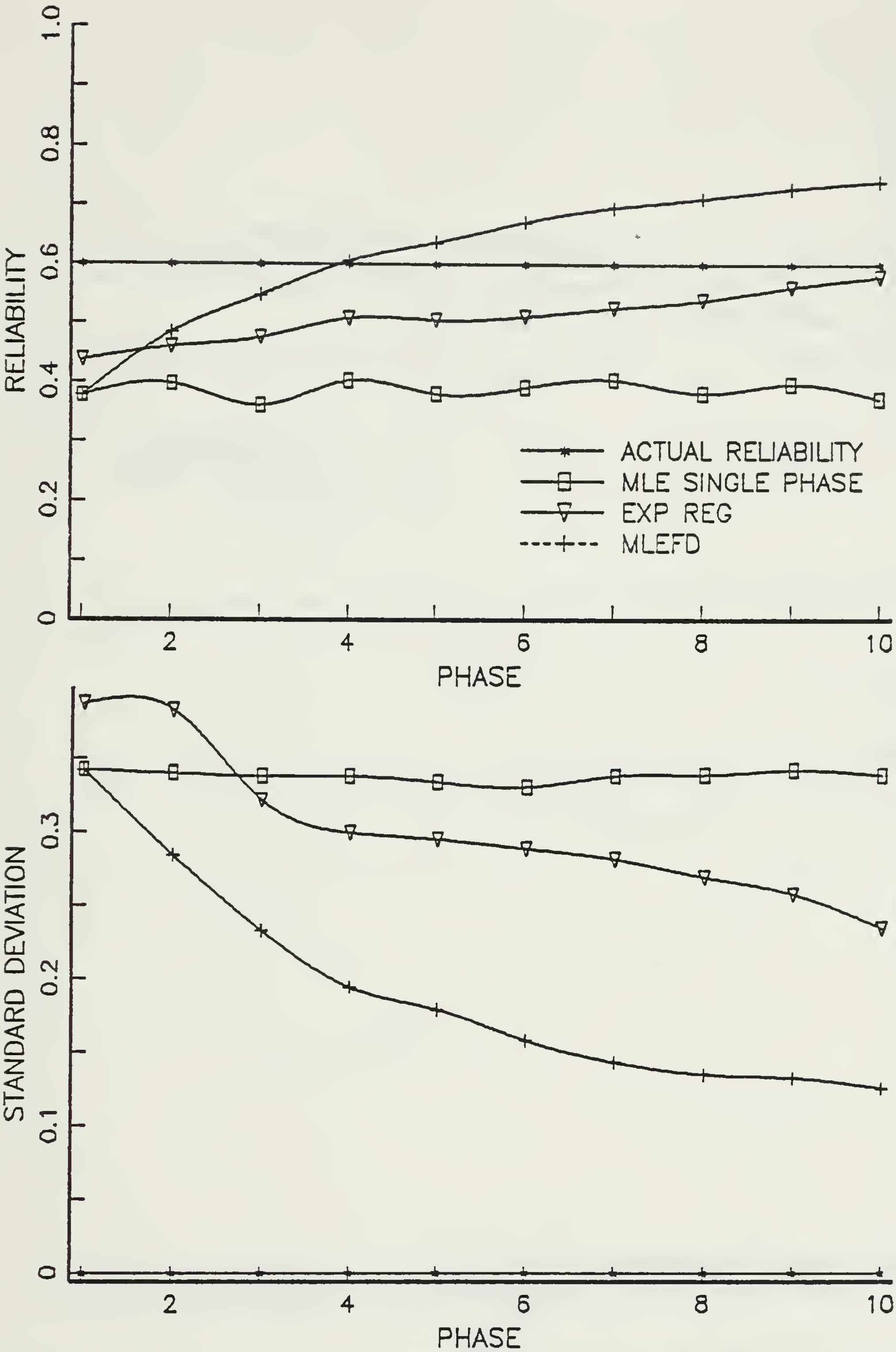

Figure 86. Pattern VII, F = .25, I = 3

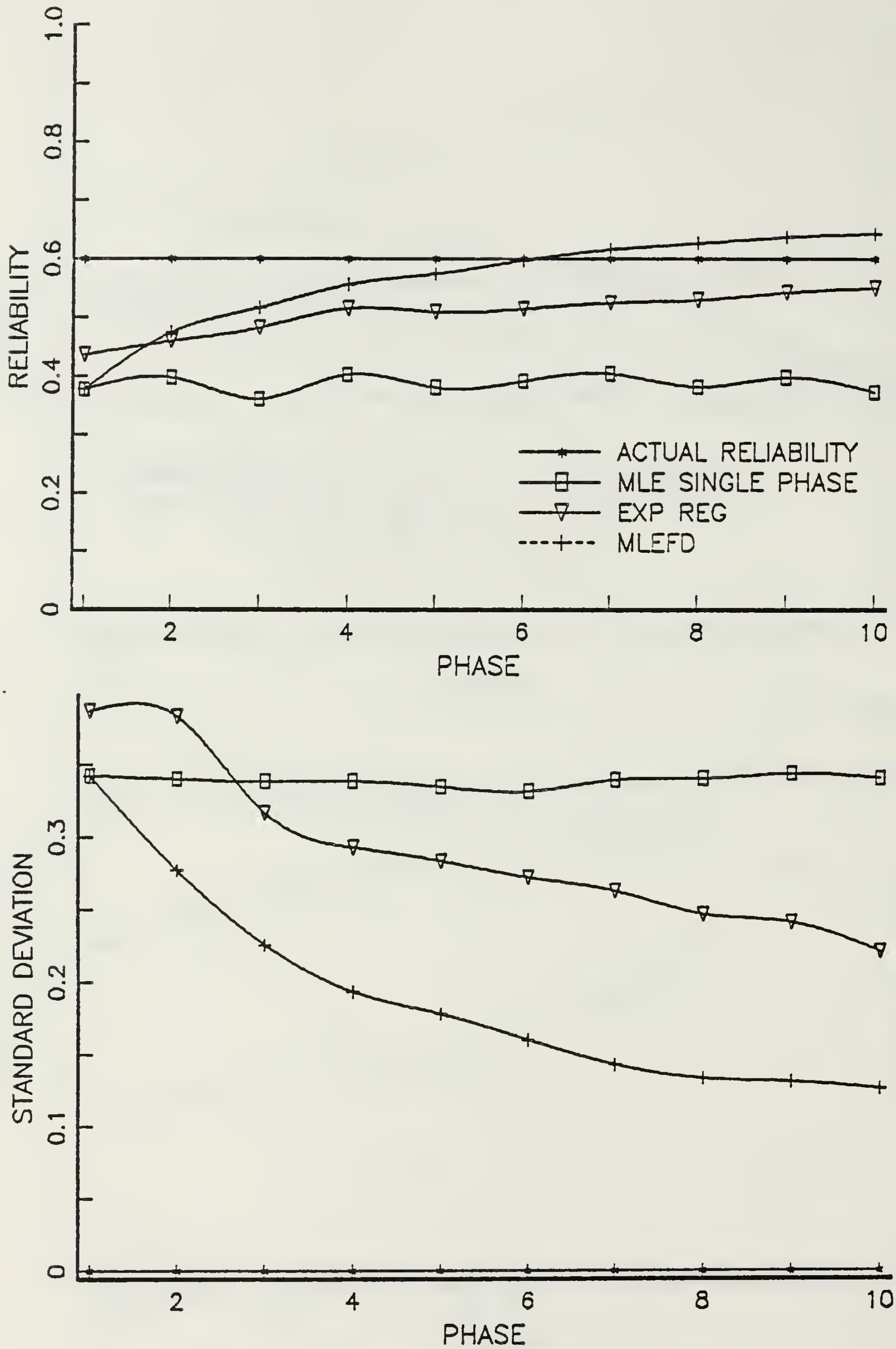

Figure 87. Pattern VII, F = .25, I = 6

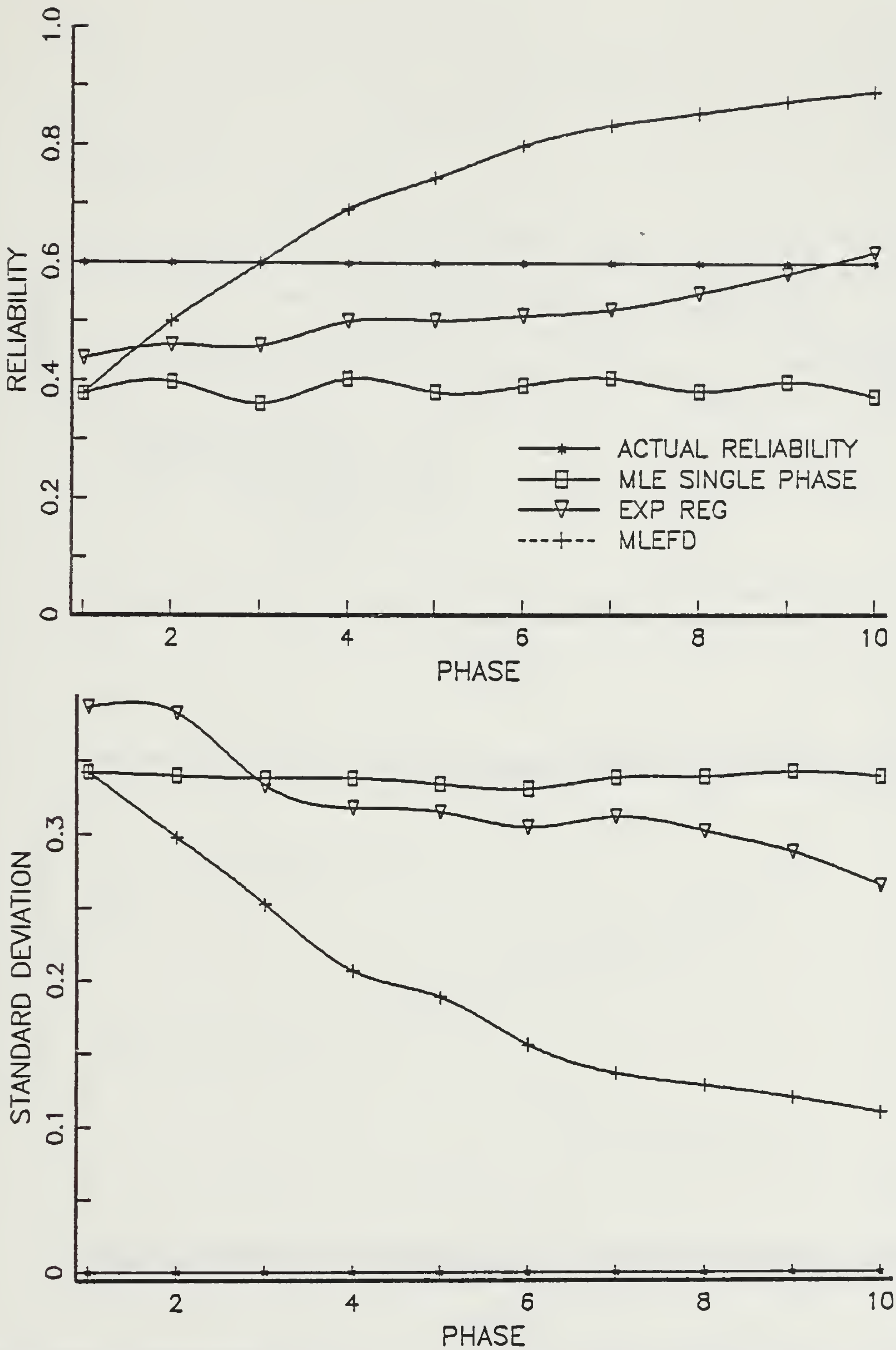

Figure 88. Pattern VII, F = .50, I = 3

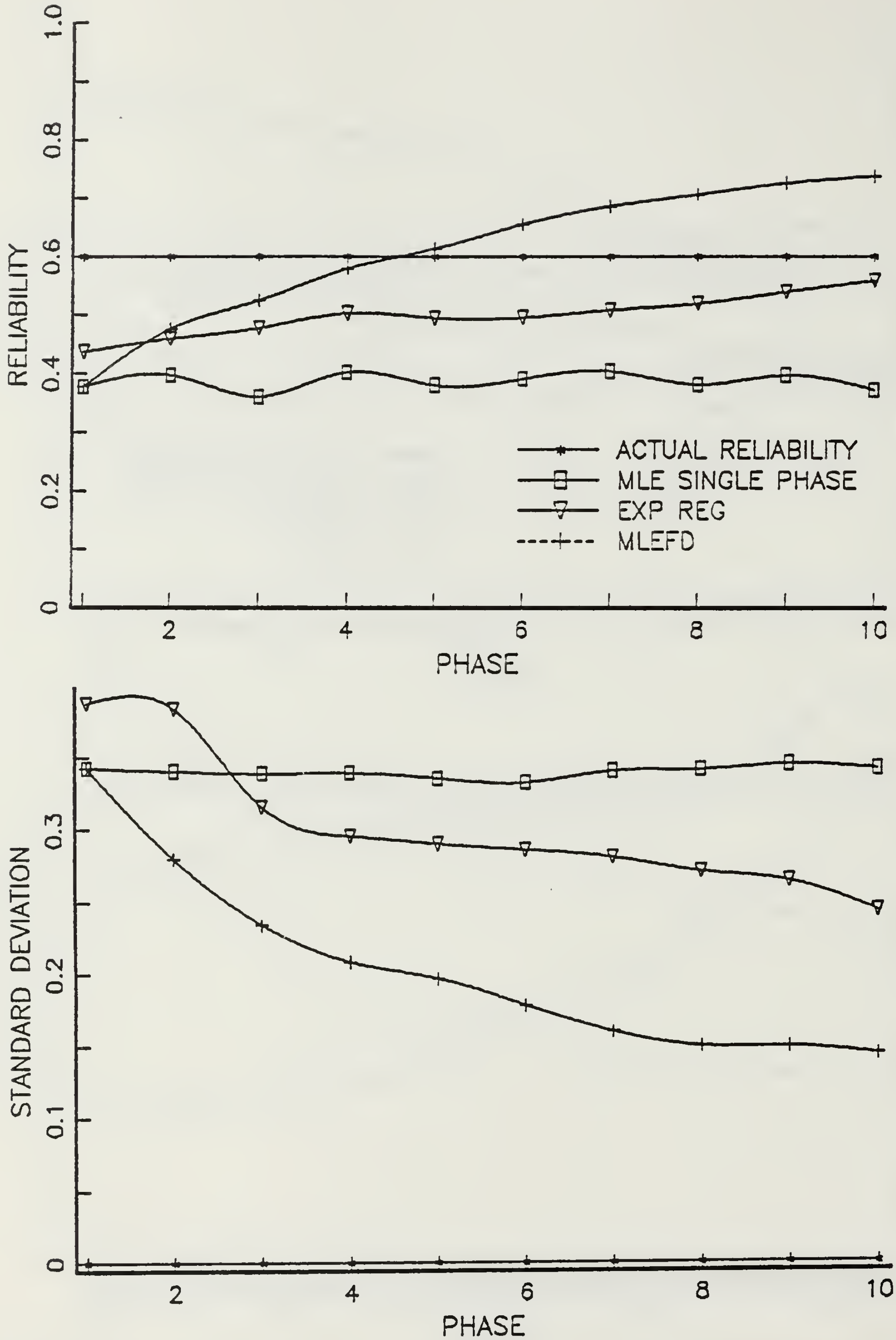

Figure 89. Pattern VII, F = .50, I = 6

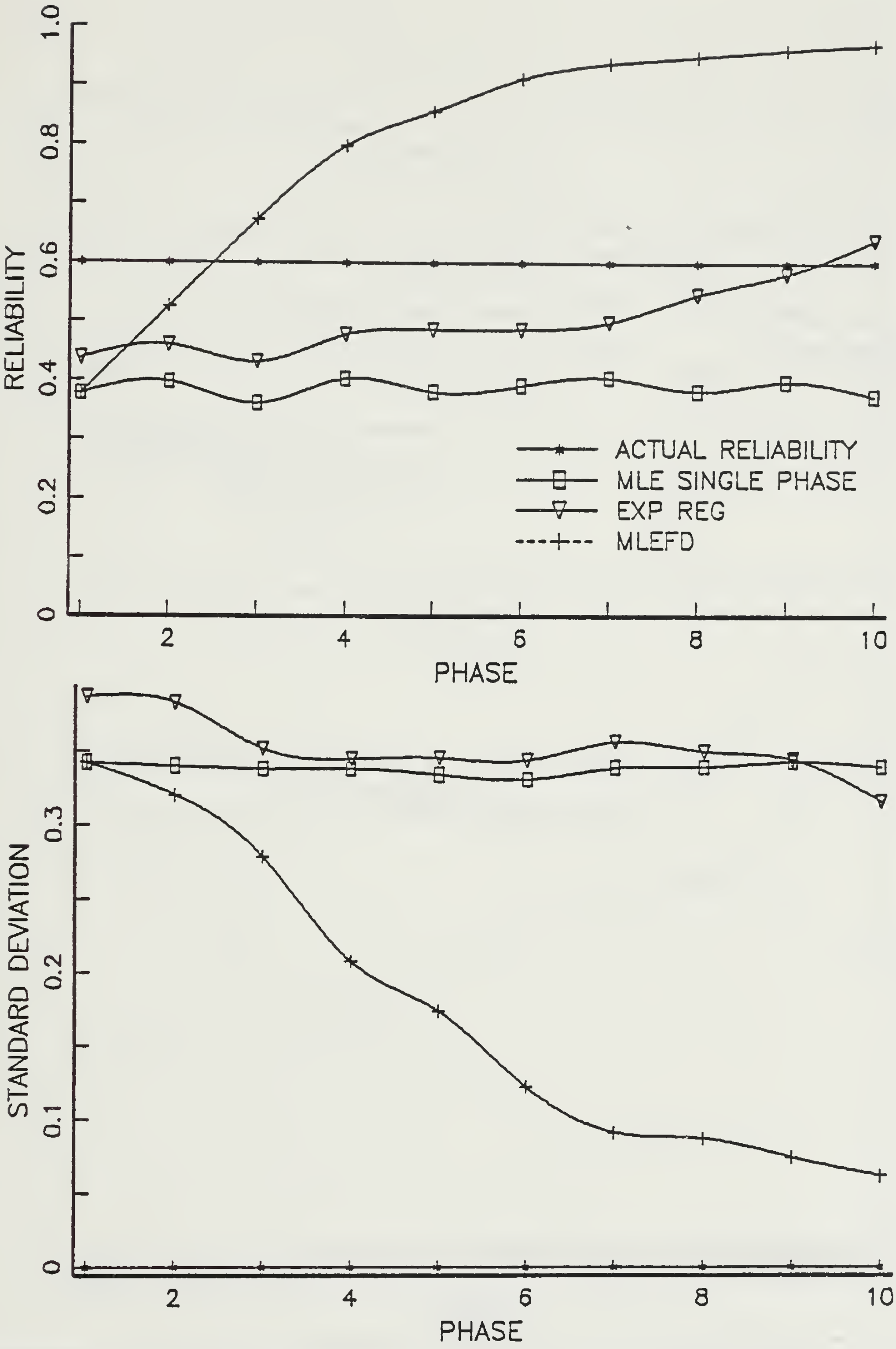

Figure 90. Pattern VII, F = .75, I = 3

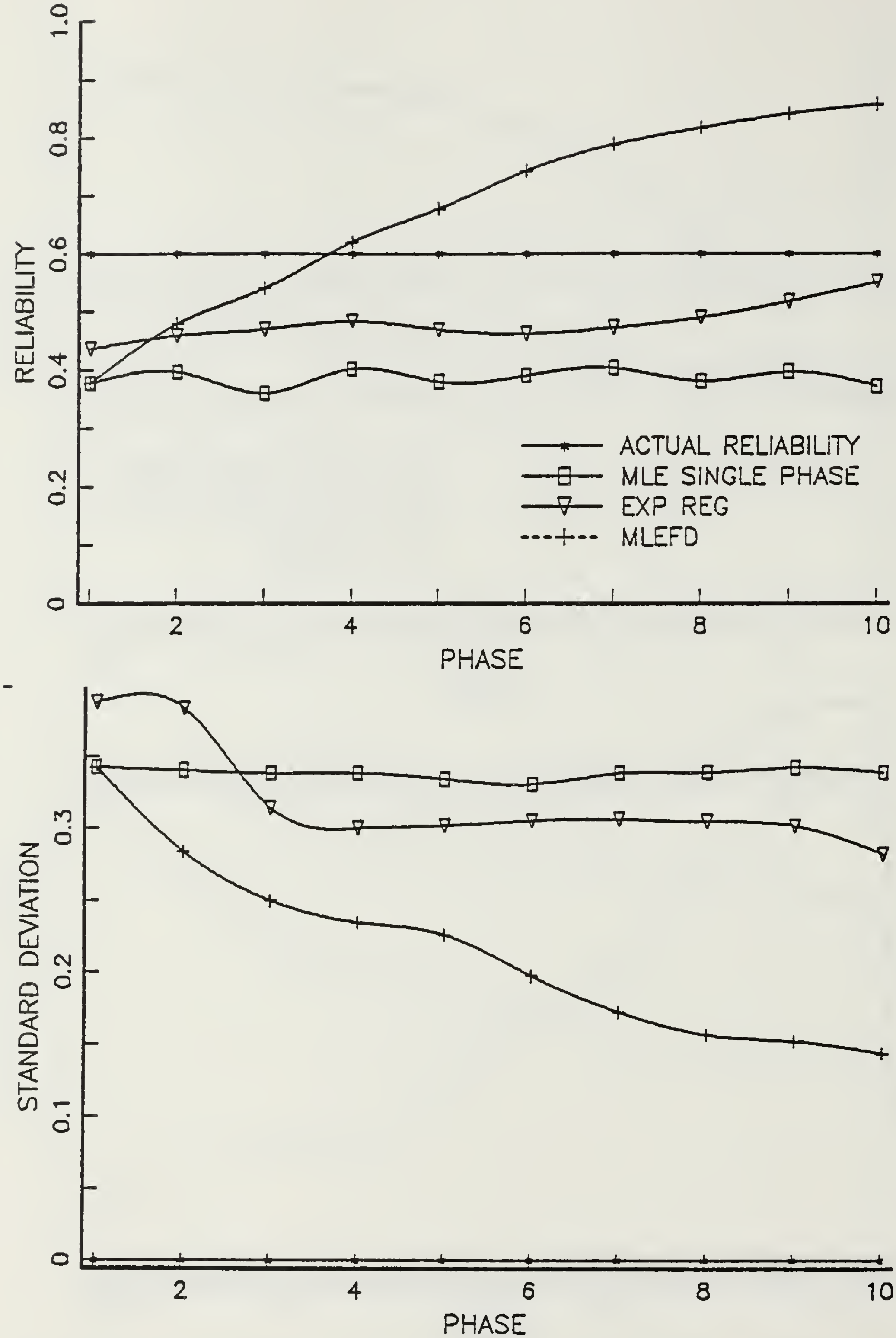

Figure 91. Pattern VII, F = .75, I = 6

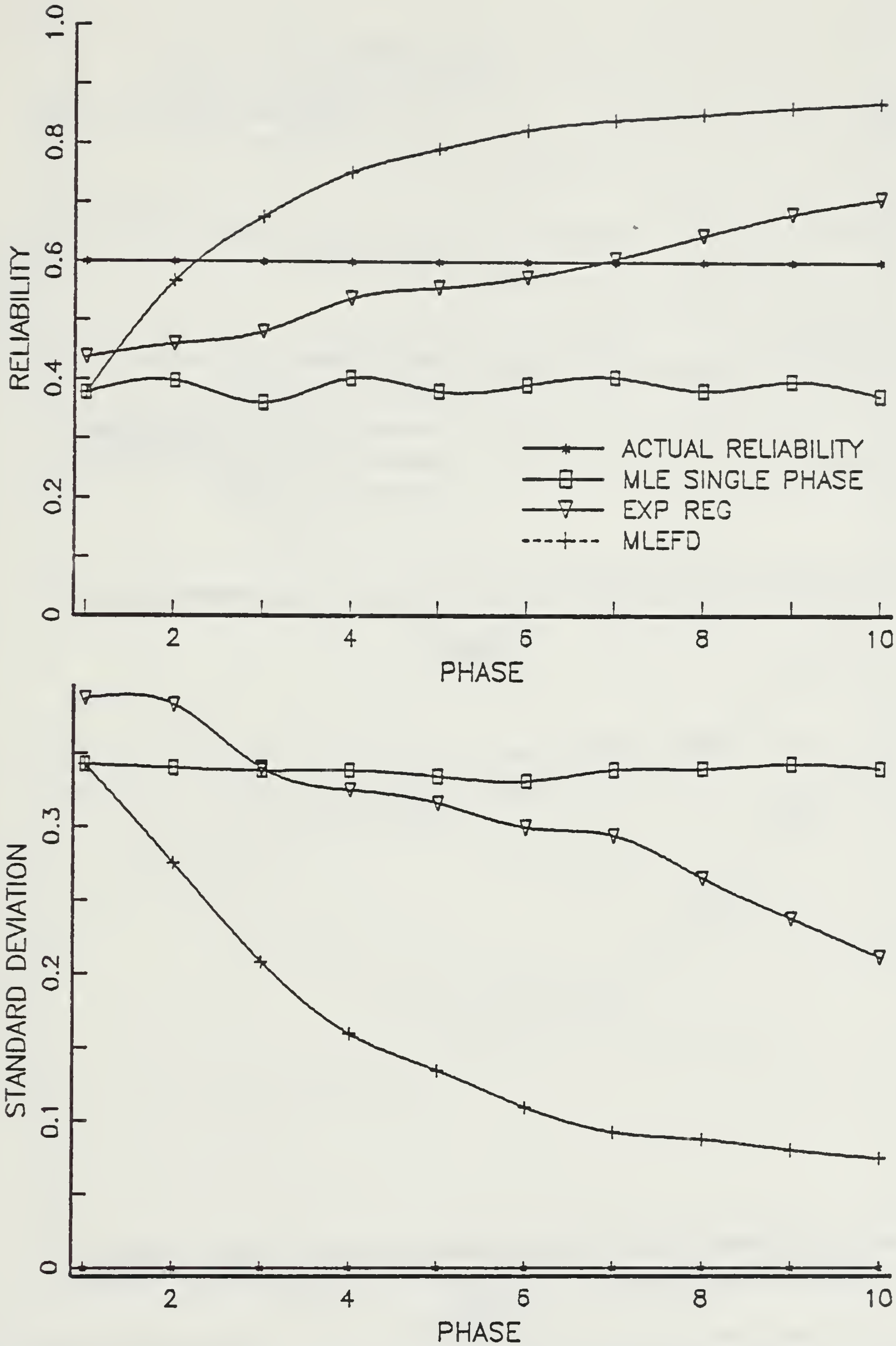

Figure 92. Pattern VII, Lloyd, CI = .8

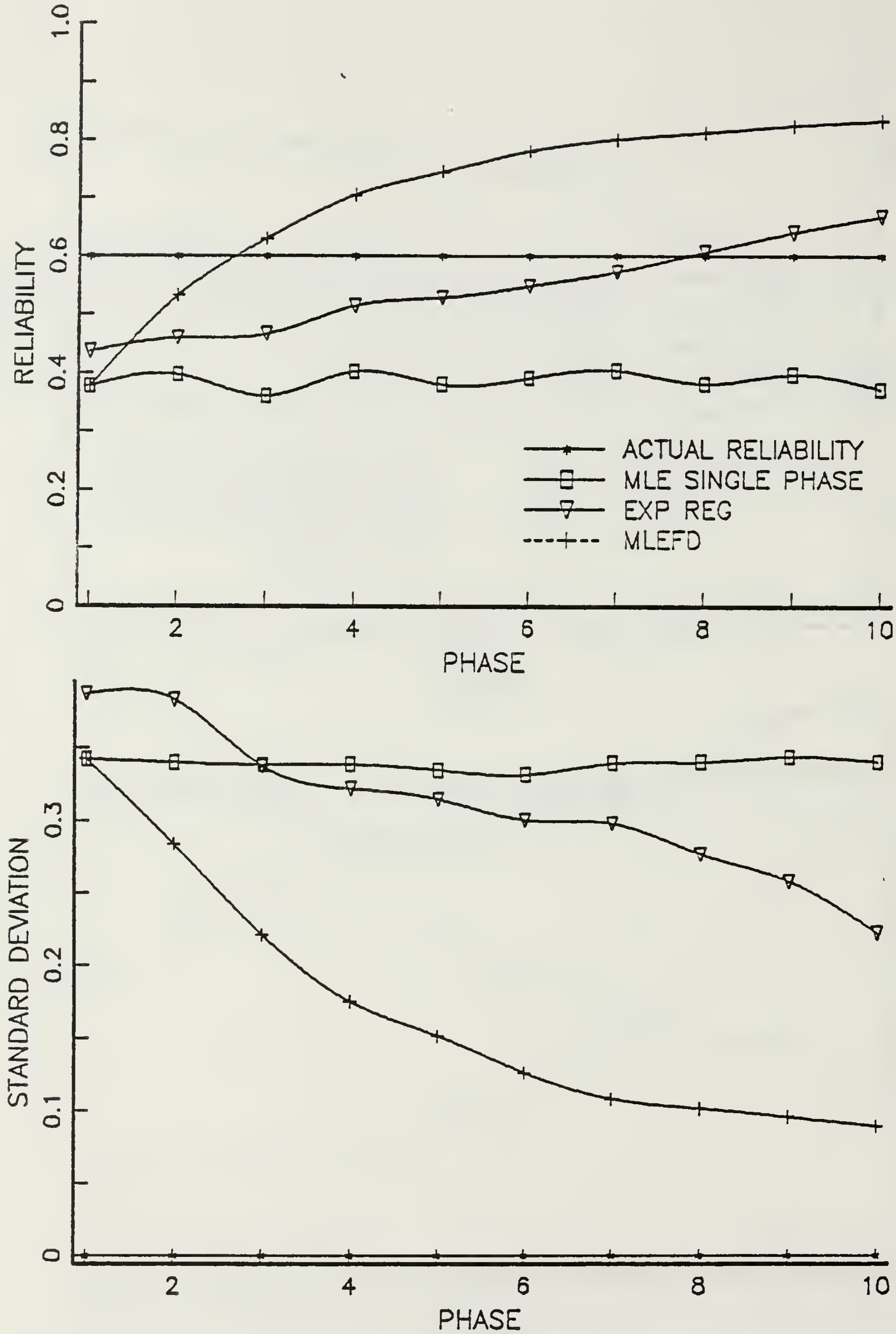

Figure 93. Pattern VII, Lloyd, CI = .9

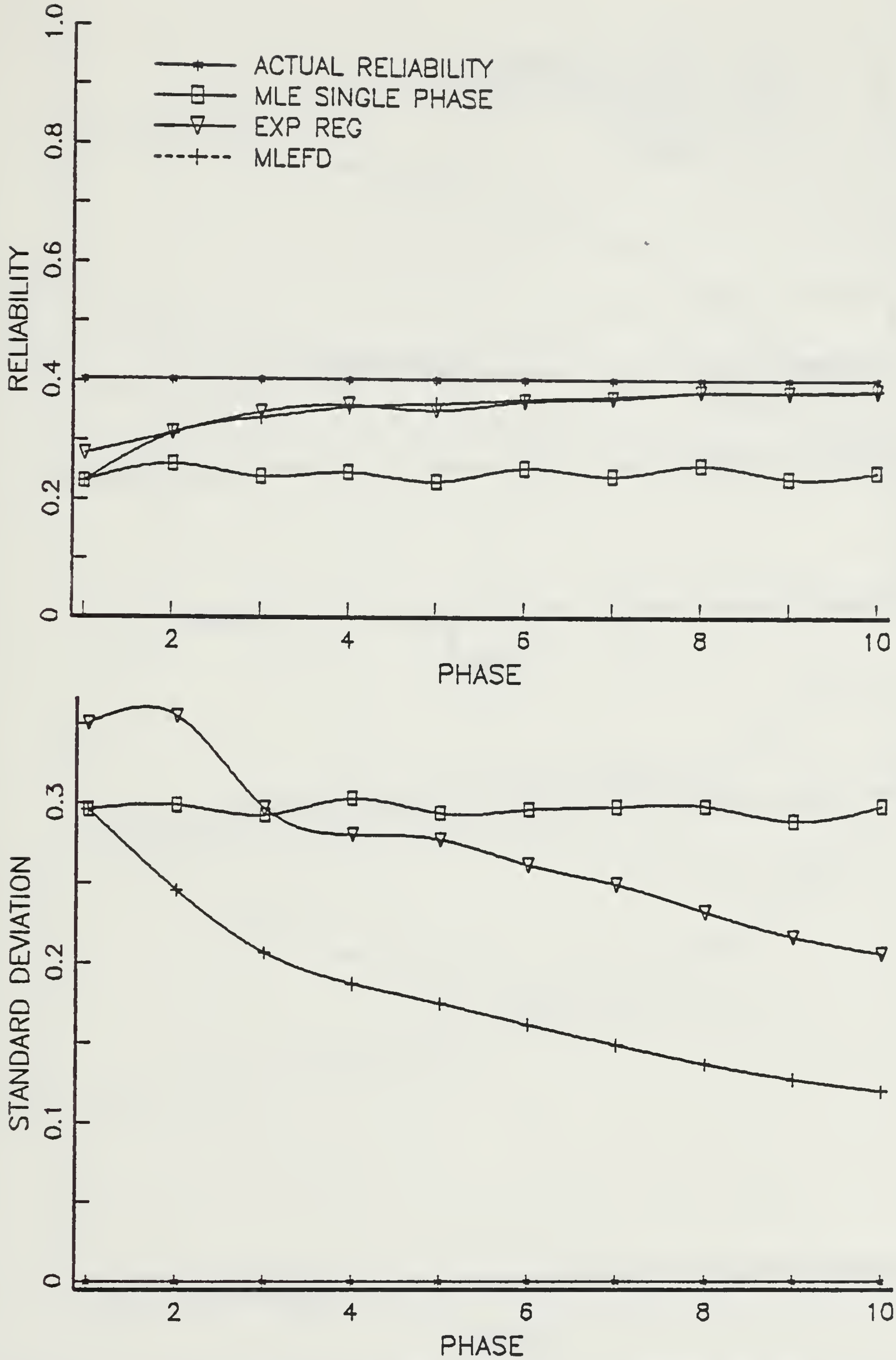

Figure 94. Pattern VIII, No Discounting

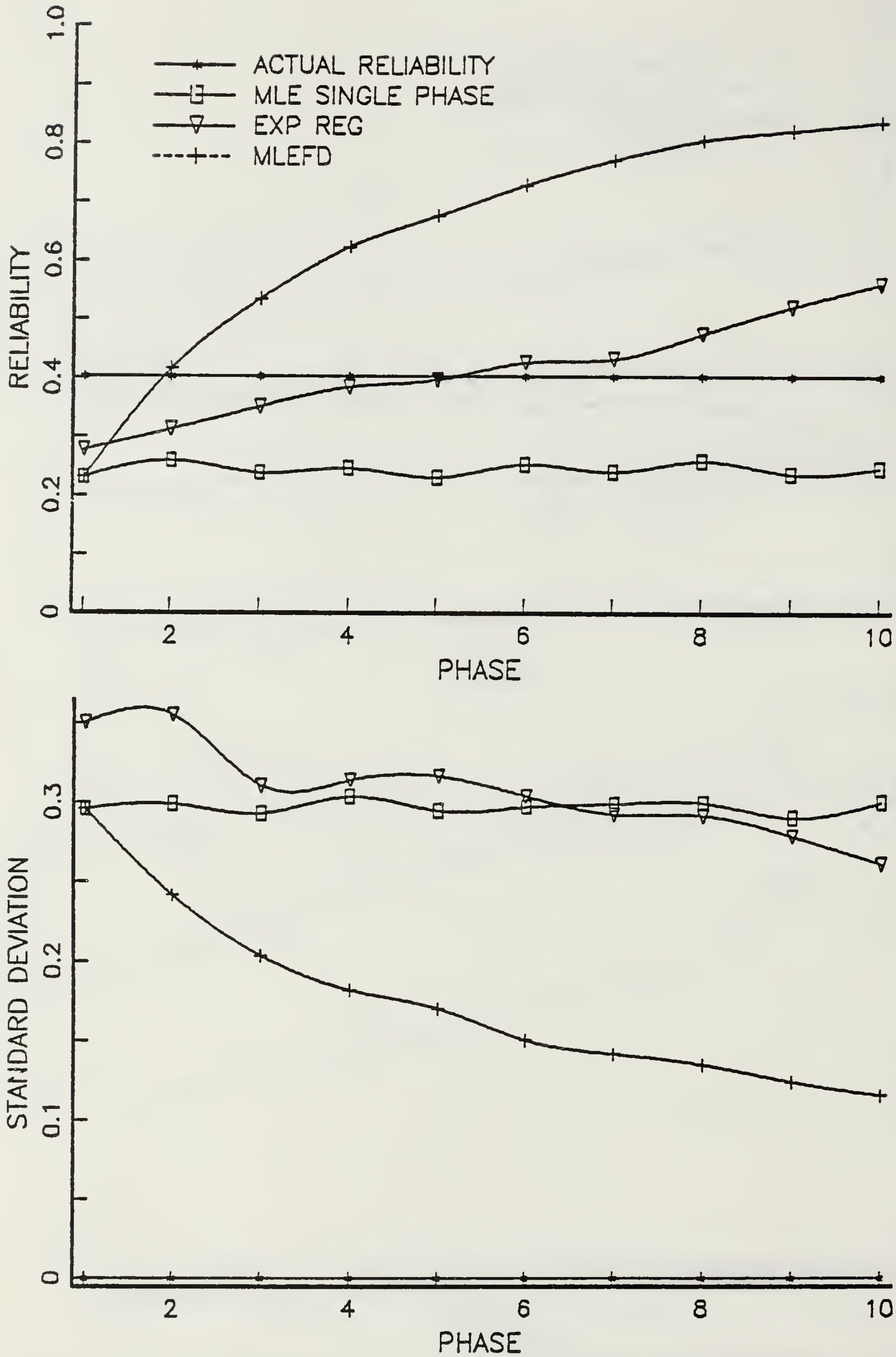

Figure 95. Pattern VIII, F = .25, I = 1

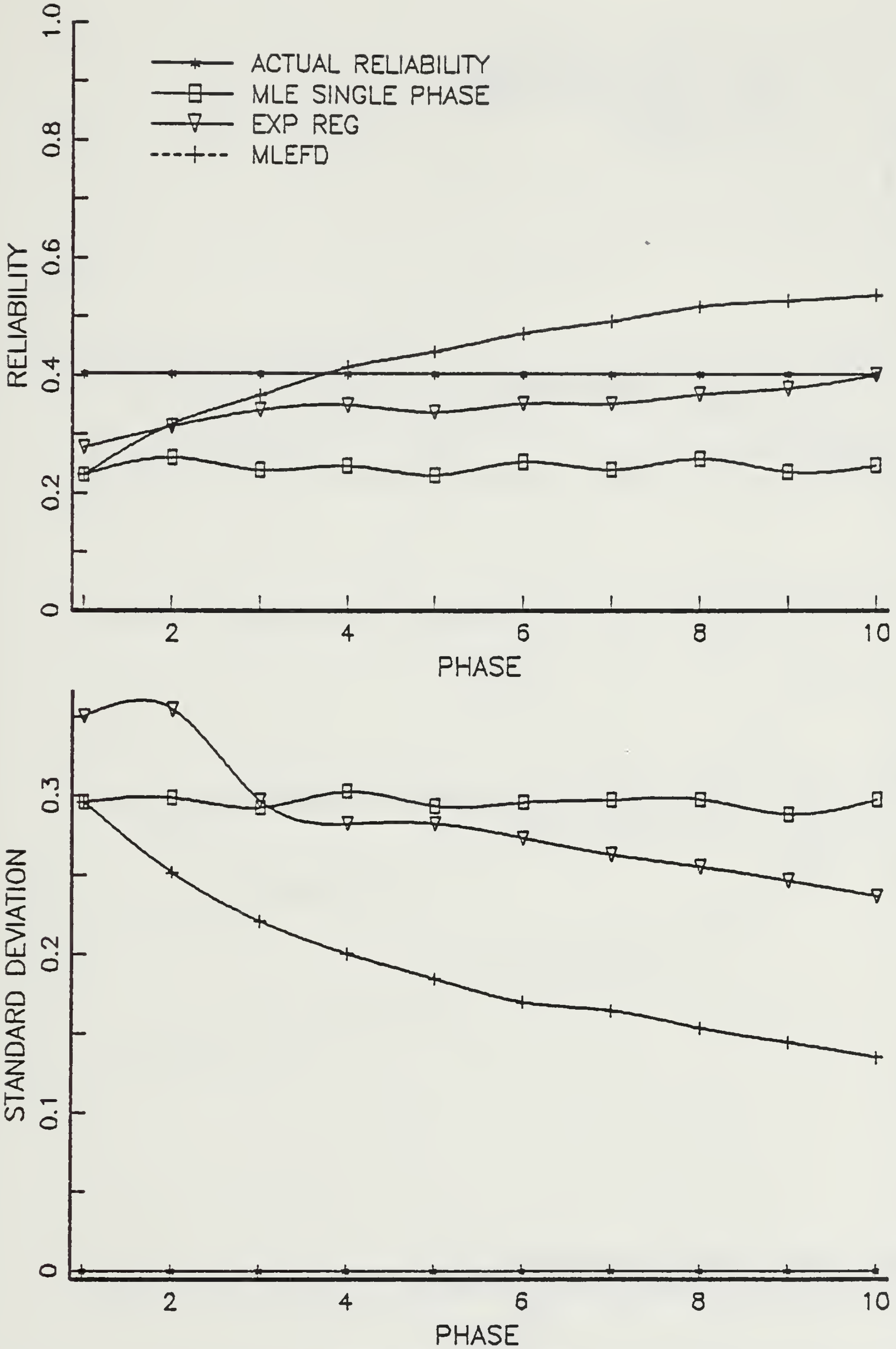

Figure 96. Pattern VIII, F = .25, I = 3

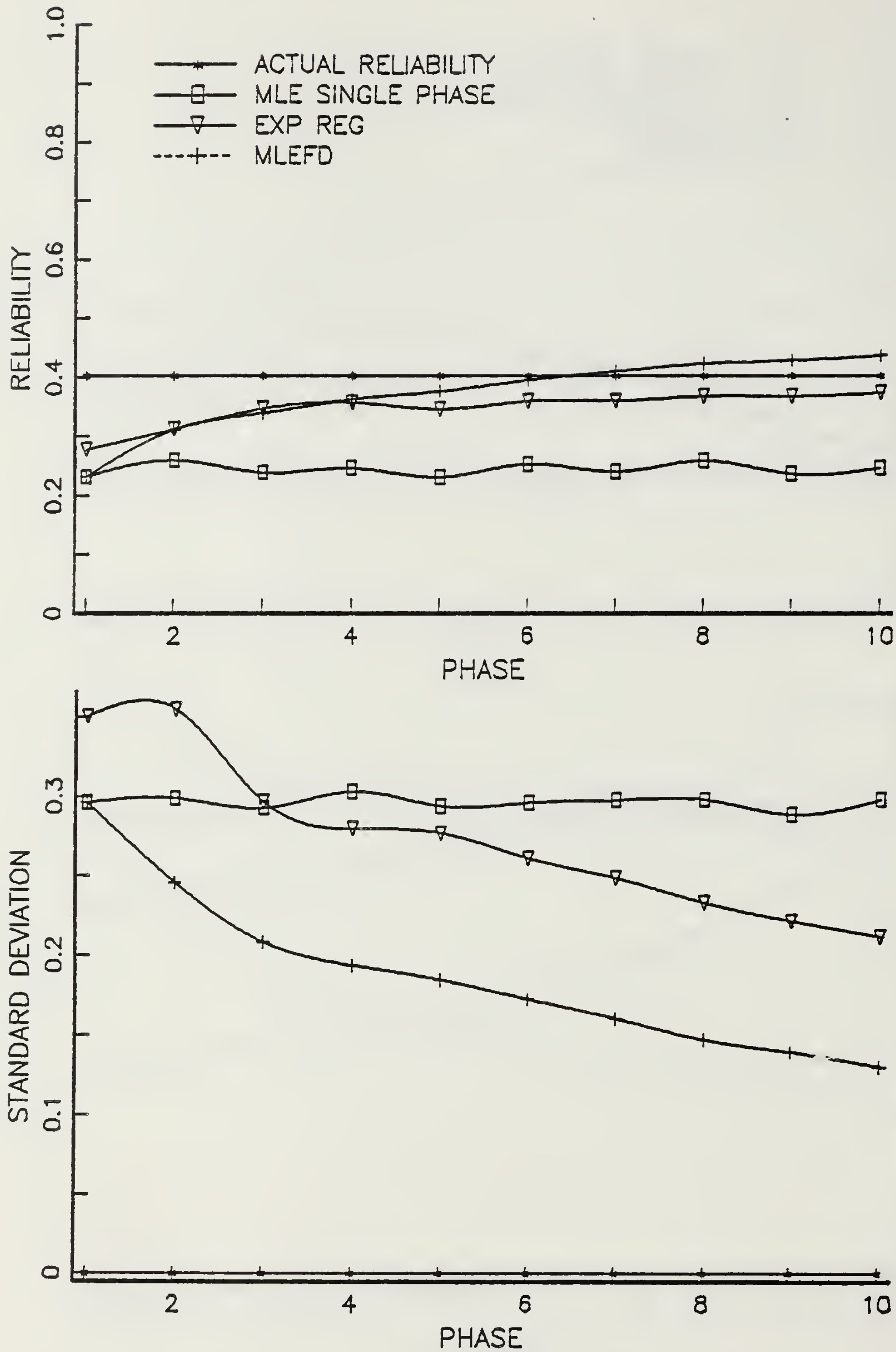

Figure 97. Pattern VIII, F = .25, I = 6

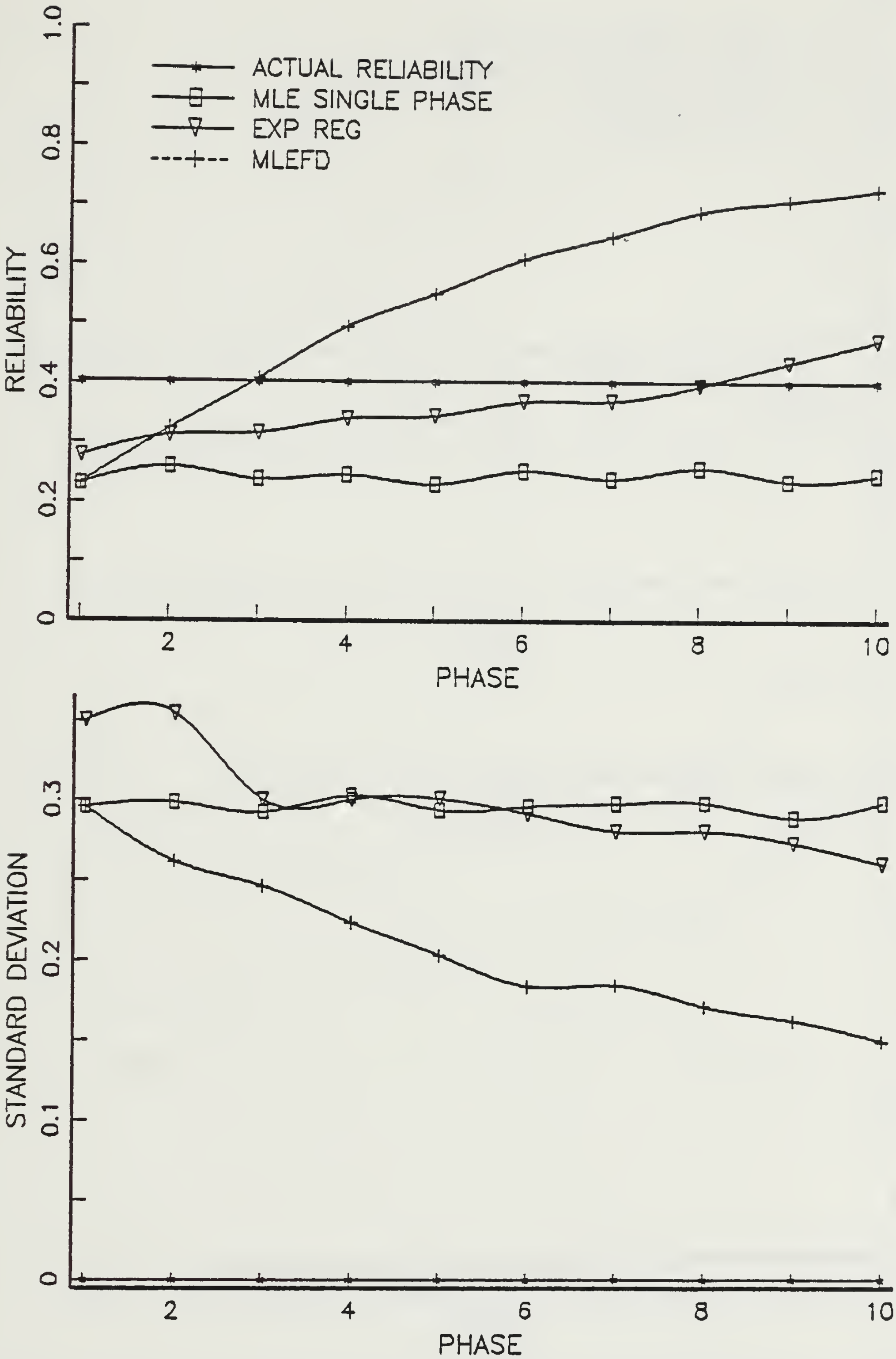

Figure 98. Pattern VIII, F = .50, I = 3

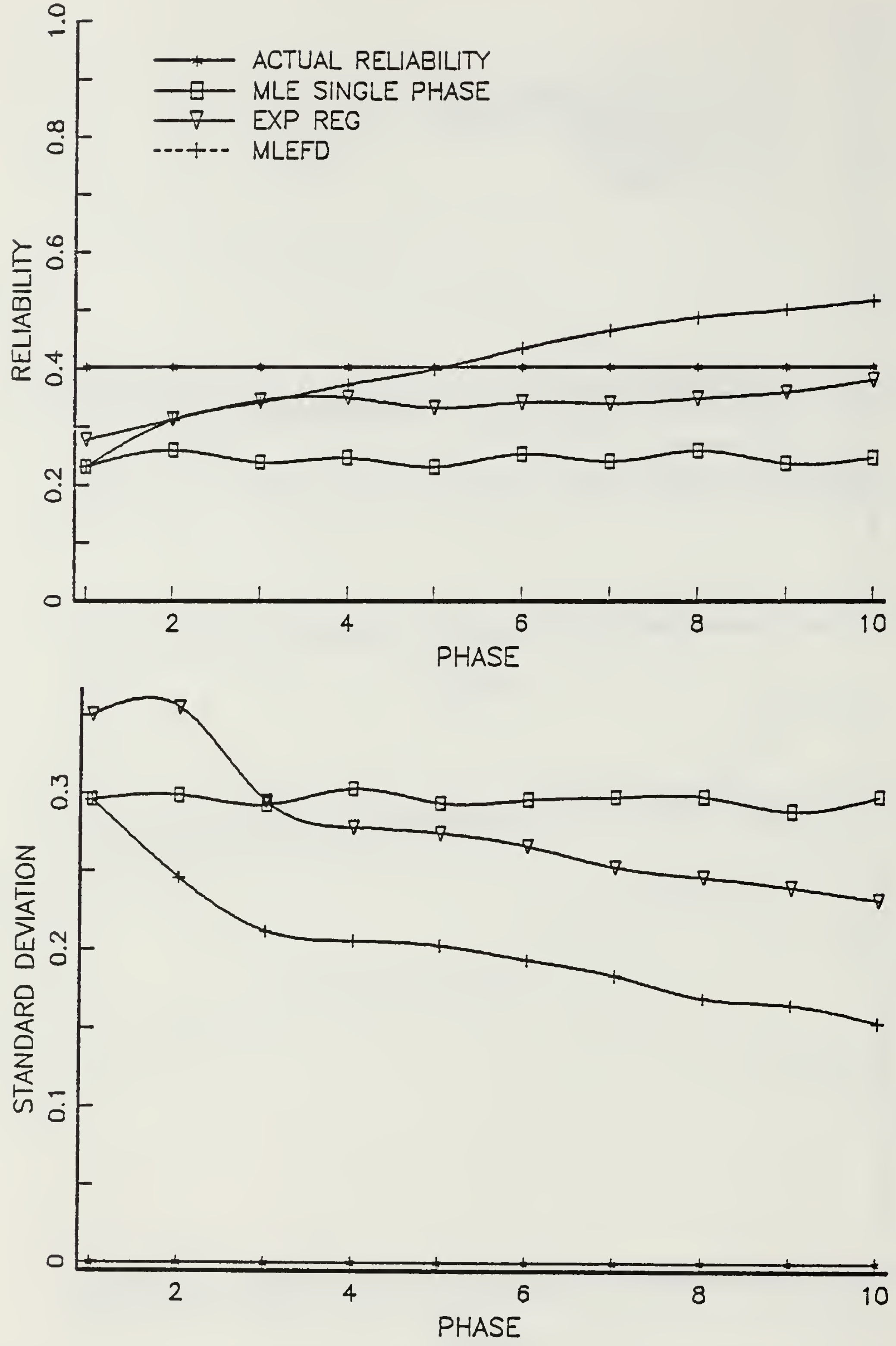

Figure 99. Pattern VIII, F = .50, I = 6

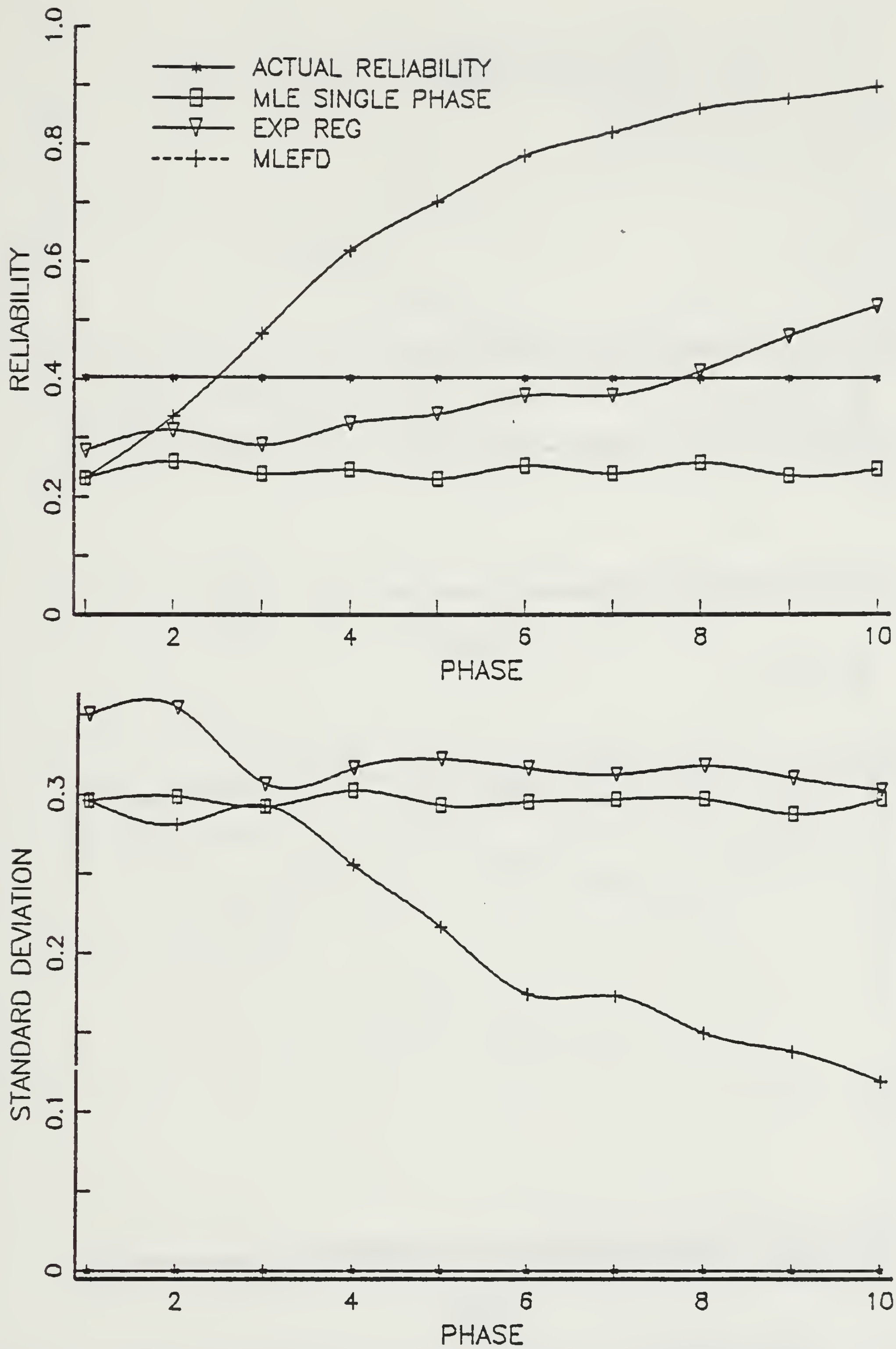

Figure 100. Pattern VIII, F = .75, I = 3

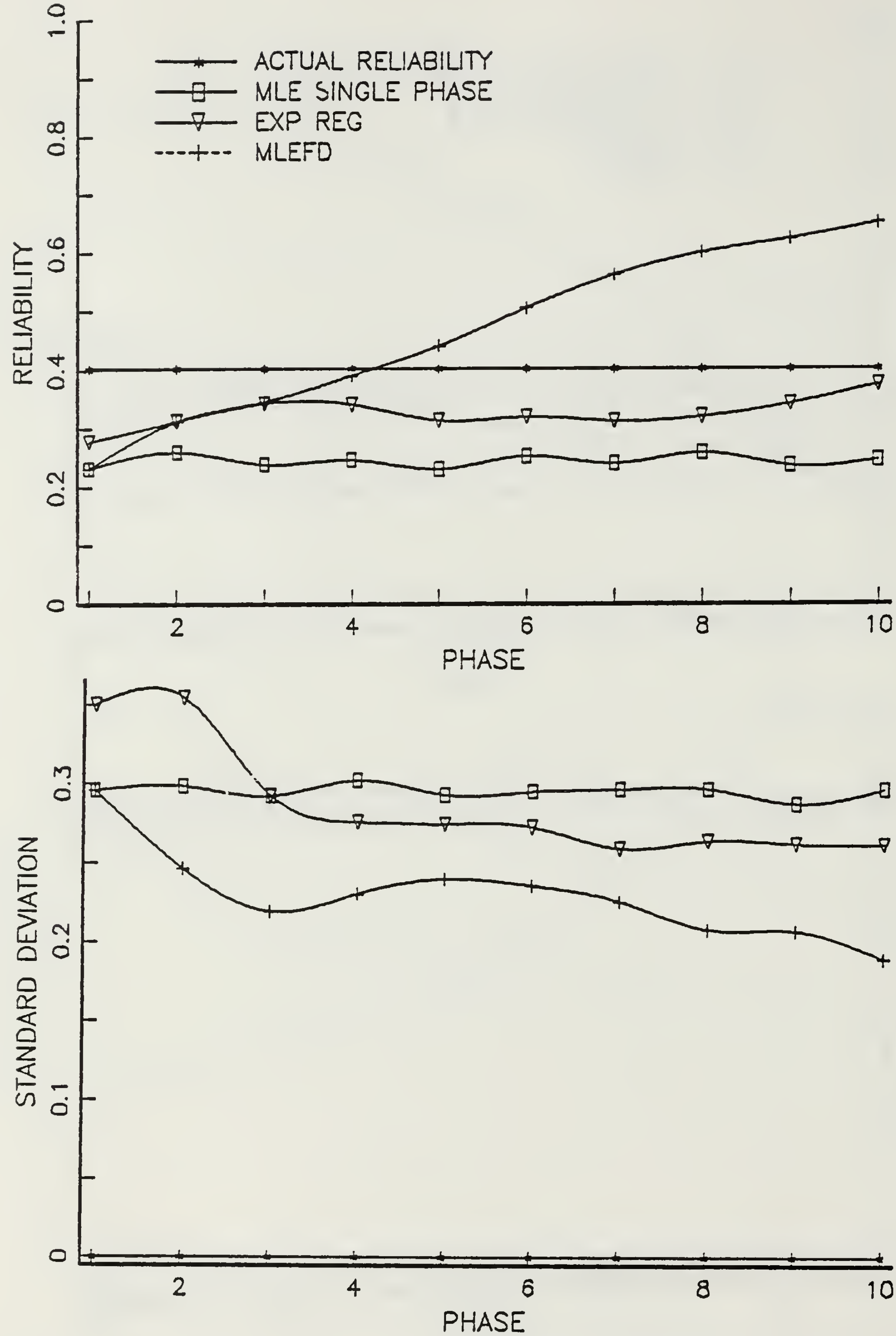

Figure 101. Pattern VIII, F = .75, I = 6

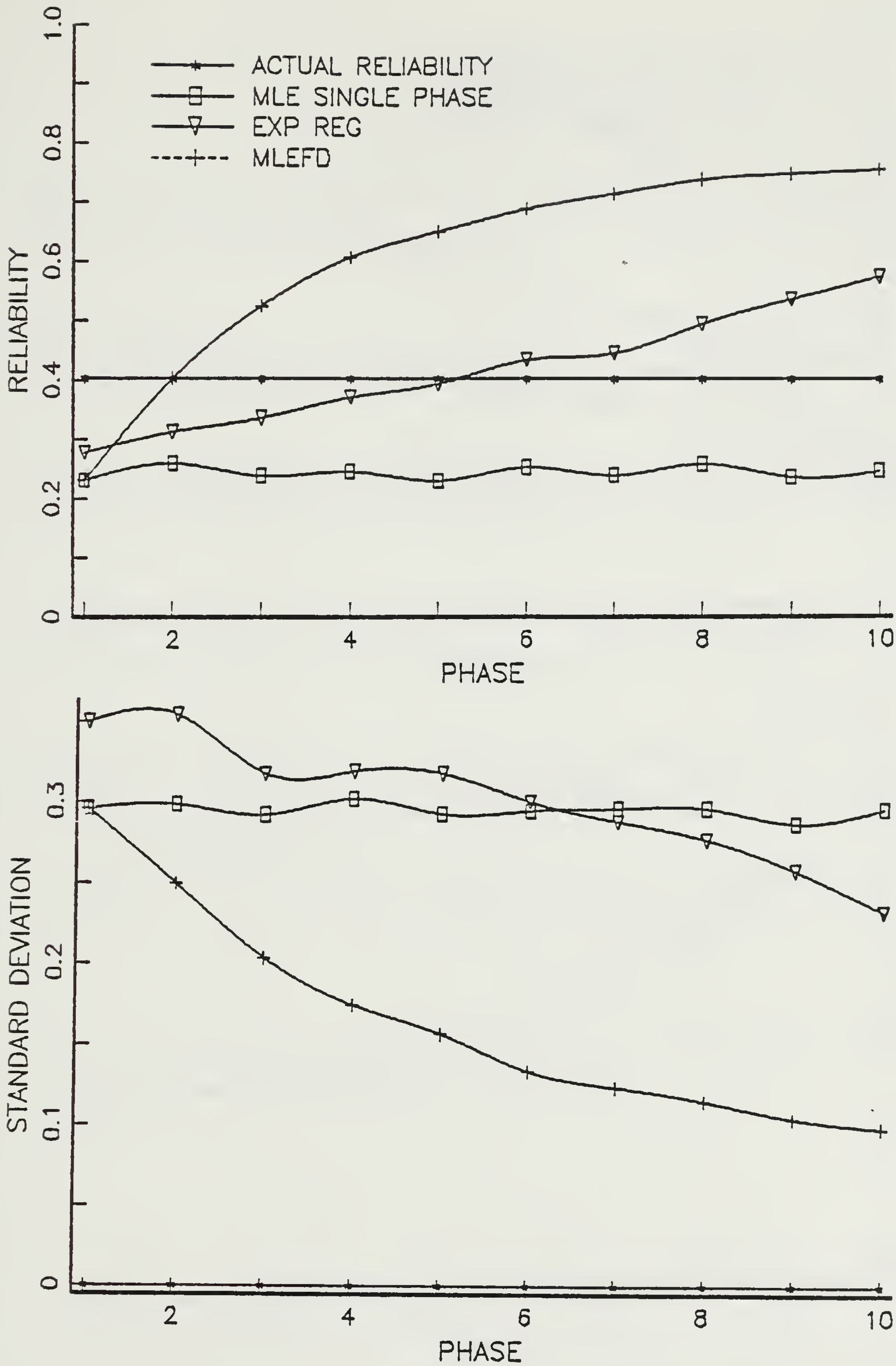

Figure 102. Pattern VIII, Lloyd, CI = .8

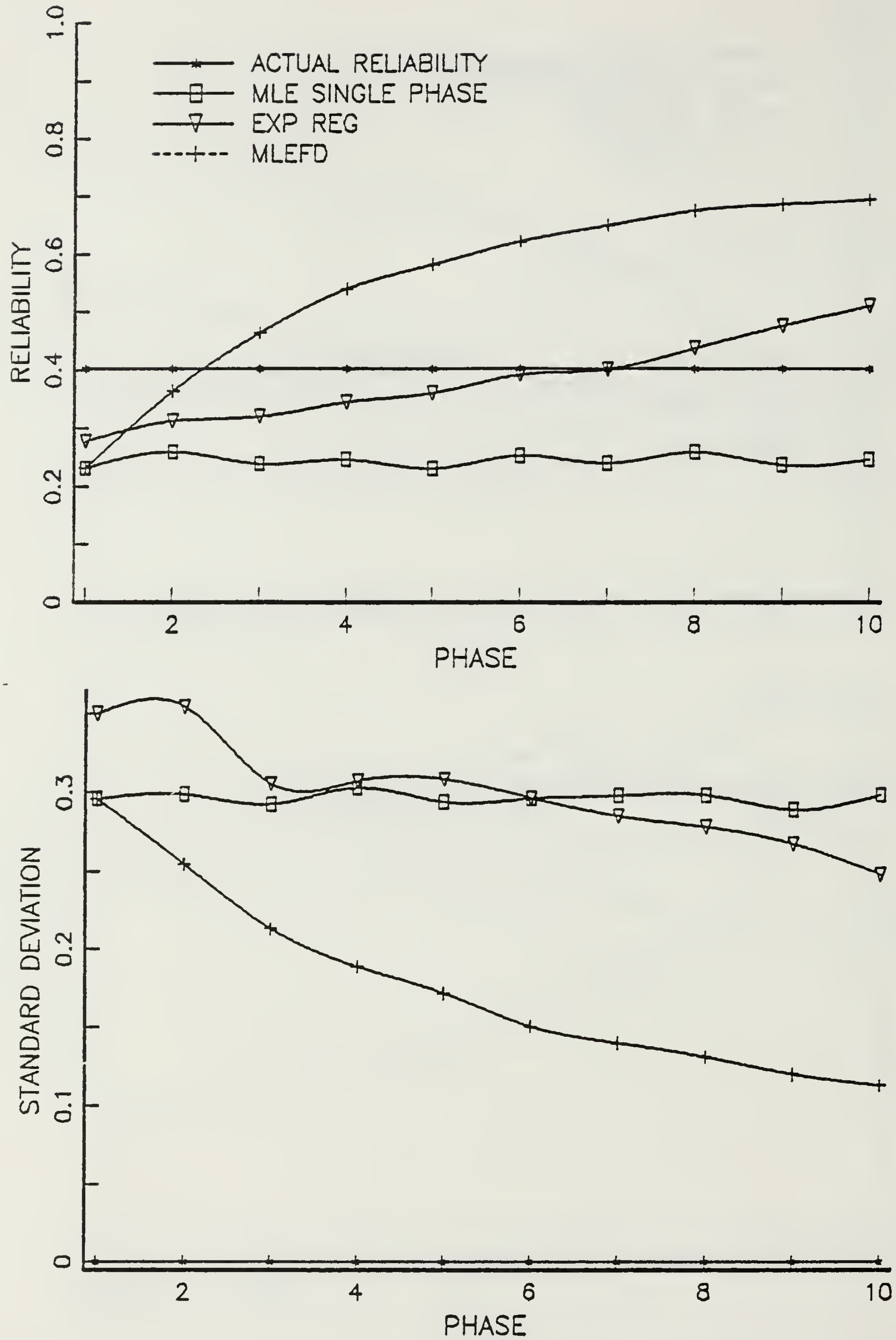

Figure 103. Pattern VIII, Lloyd, CI = .9

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